Aerodynamics II

Performance of a Supersonic Airfoil

Group Project

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Abstract

The performance of a supersonic wing can be analyzed using linear theory or shock-expansion technique. This project was conducted to become more familiar with these two differing techniques, and to analyze the performance of a diamond shaped airfoil section at a given altitude of 44000 ft, a weight of 31500 lb, a thickness ratio of 0.05, and a wing area of 300 ft^2. Lift, wave drag, and pitching moment coefficients were found as functions of Mach numbers, ranging from Mach 1.4 to 2.8 in increments of 0.2. The data acquired from the two techniques was very consistent with fluctuation on the order of 0.001 for the wave drag. The Mach number at which the wave drag is minimum was found to be 2.8. Pressure and temperature distributions on the surface of the airfoil were plotted at this Mach number. Maximum temperature and pressure were found on section 3 of the airfoil (see Figure 1) with values of 445.2 $\frac{lb}{ft^2}$ and 464.4 $\frac{lb}{ft^2}$, and 426.8 R and 432.6 R, using linear theory and shock-expansion technique respectively. Minimum temperature and pressure were found on section 2 of the airfoil with values of 204.0 $\frac{lb}{ft^2}$ and 220.7 $\frac{lb}{ft^2}$, and 341.5 R and 349.3 R, using linear theory and shock-expansion technique respectively. The minimum wave drag was found to be 0.00610 and 0.00615, the minimum lift coefficient was found to be 0.0589 and 0.0595, using linear theory and shock-expansion technique respectively. The minimum pitching moment coefficient about half-cord equals to zero using linear theory and was found to be 0.00246 using shock-expansion technique.

Contents

List of Figures

List of Tables

1 Introduction

The objective of this project is to analyze the performance of a supersonic wing with a symmetric diamond shaped airfoil section. The wing is analyzed using both the linear theory and shock-expansion technique. The shock-expansion technique calculates pressures on the airfoil surface using oblique shocks and Prandtl-Meyer expansions. Lift, wave drag, and pitching moment coefficients of the airfoil are calculated as functions of Mach numbers in increments of 0.2 (from M =1.4 to M = 2.8). The pressure and temperature distributions across the airfoil are plotted for the Mach number at which wave drag is minimum. Figure 1 shows a diamond-wedge airfoil in supersonic flow. Throughout this report, indicated subscripts of calculated parameters match the surface numbering in this figure.

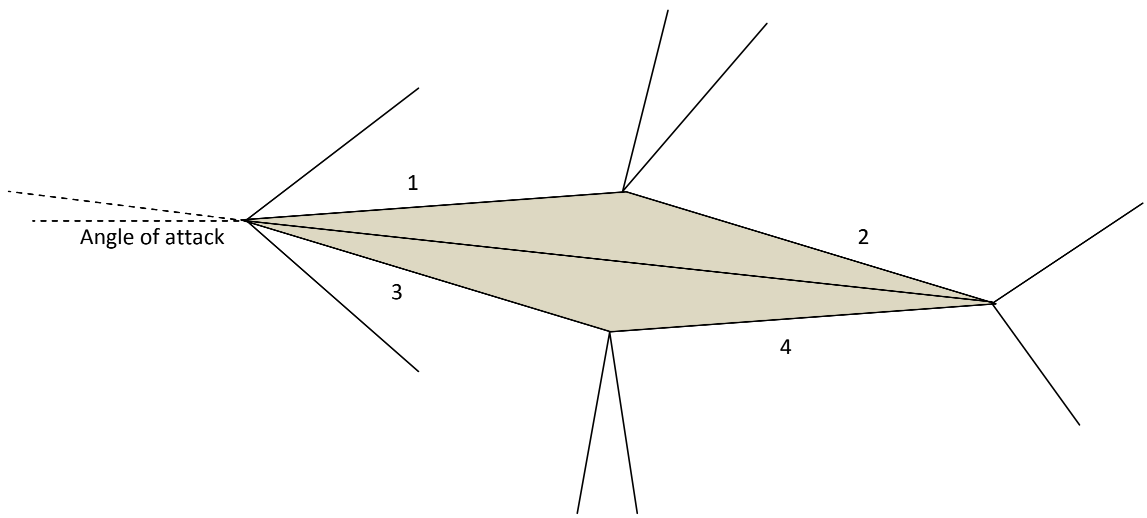


Figure 1: Symmetric diamond shaped airfoil

1.1 Document Overview

1.2 Nomenclature

Following commonly used abbreviations are used throughout this report. If not otherwise stated, English engineering units are used.

Table 1: Nomenclature

C_L	Lift coefficient	
C_d	Drag coefficient	
C_p	Pressure coefficient	
$C_{m,\frac{1}{2}c}$	Pitching Moment coefficient	
W	Weight	lbf
γ	Specific heat	
p_∞	Free stream pressure	$\frac{lb}{ft^2}$
q_∞	Dynamic pressure	$\frac{lb}{ft^2}$
M_∞	Free stream Mach number	
M_n	Normal Mach number	
$\nu(M)$	Prandtl-Meyer function	
S	Wing Area	ft^2
c	Cord length	ft
α	Angle of attack	radians
$\Delta\theta$	Deflection Angle	degrees
β	Wave angle	degrees
δ_w	half wedge angle	radians

2 Theory

Two different theories are used within this report. An overview of each is given in this section. X_j indicates a calculated value X for a corresponding Mach number M_j.

2.1 Linear Theory

The linearized supersonic flow theory is an approximation theory assuming small perturbations. It can be used for supersonic speeds only, rather than transonic or hypersonic. The theory summarized below is very useful for estimating the lift and wave drag of supersonic airfoils.
The lift coefficient can be calculated using equation (1) with a given set of information.

$$C_{L,j} = \frac{W}{q_{\infty,j} S} \tag{1}$$

With the lift coefficient found, the angle of attack is found using equation (2).

$$\alpha_j = C_{L,j} \frac{\sqrt{M_{\infty,j}^2 - 1}}{4} \tag{2}$$

With the angle of attack, the drag coefficient is calculated using equation (3).

$$C_{d,j} = \frac{4\alpha_j^2}{\sqrt{M_{\infty,j}^2 - 1}} + \frac{2}{\sqrt{M_{\infty,j}^2 - 1}} \int_0^1 \left[(\frac{dy_u}{dx})^2 + (\frac{dy_l}{dx})^2 \right] d(\frac{x}{c}) \tag{3}$$

The pitching moment can be calculated using equation (4). However, since the airfoil is symmetric and at supersonic speed, the pitching moment about half cord will always be zero and therefore is omitted in the calculation section.

$$C_{m,\frac{1}{2}c,j} = -\frac{4\alpha_j}{\sqrt{M_{\infty,j}^2 - 1}} (\frac{1}{2} - \frac{x_0}{c}) + \frac{4\alpha_j}{\sqrt{M_{\infty,j}^2 - 1}} \int_0^1 (\frac{dy_c}{dx}) \frac{x - x_0}{c} d(\frac{x}{c}) \tag{4}$$

The pressure and temperature distribution is calculated for the Mach number at which the wave drag is minimum. Using equation (5) and (6), respectively. Both the upper and lower surface have two different coefficients for the index corresponding to $\min\{C_{d,j}\}$.

$$C_{p,u,i,j} = \frac{2\theta_{u,i,j}}{\sqrt{M_{\infty,j}^2 - 1}} \tag{5}$$

where

$$\theta_{u,i,j} = \frac{dy_{u,i}}{dx} - \alpha_j$$

$$C_{p,l,i,j} = \frac{2\theta_{l,i,j}}{\sqrt{M_{\infty,j}^2 - 1}} \tag{6}$$

where

$$\theta_{l,i,j} = -\frac{dy_{u,i}}{dx} + \alpha_j$$

With the pressure coefficients calculated, the pressure of each surface is calculated by equation (7).

$$p_{i,j} = C_{p,i,j} * q_{\infty,j} + p_\infty \tag{7}$$

Since flow is isentropic, temperature is related to pressure by equation (8).

$$T_{i,j} = T_\infty \left(\frac{p_{i,j}}{p_\infty} \right)^{\frac{\gamma - 1}{\gamma}} \tag{8}$$

2.2 Shock Expansion Technique

Shock Expansion Technique (SET) can be divided into two sections, being Prandtl-Meyer expansion-wave theory and oblique shock theory. In order to calculate the lift and drag coefficients, the pressure coefficients on each surface of the airfoil must be calculated. For surface one and three, the angle is determined considering the slope of the surface versus the angle of attack. For surface two and four, the angel is measured from the stream direction of the previous surface to the local surface slope. Note that calculated angles of attack from the linear theory are also used for this technique.

2.2.1 Prandtl-Meyer expansion-wave theory

For a given Mach number approaching a convex corner as displayed in figure 2, the corresponding Mach angle can be found applying the Prandtl-Meyer function [1], as stated in equation (9).

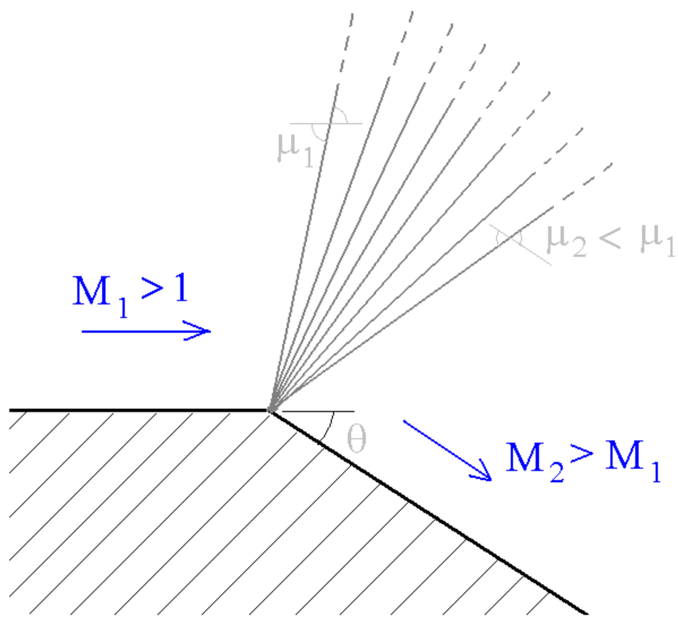


Figure 2: Prandtl-Meyer expansion fan [3]

$$\nu(M_{n,j}) = \sqrt{\frac{\gamma+1}{\gamma-1}} \arctan \sqrt{\frac{\gamma-1}{\gamma+1}(M_{n,j}^2 - 1)} - \arctan \sqrt{M_{n,j}^2 - 1} \tag{9}$$

The resulting Mach number can be found using numerical methods after applying equation (10) where $\Delta\theta$ is the expansion angle.

$$\nu(M_{n+1,j}) = \nu(M_{n,j}) + \Delta\theta_j \tag{10}$$

Equations (11) and (12) can then be used to find the pressure and temperature at the selected surface. When using expansion technique for more than one successive surface, simply multiply the equations with the second ratio.

$$p_{n+1,j} = p_{n,j}\frac{p_{n+1,j}}{p_{n,j}} = p_{n,j}\left(\frac{1+\frac{\gamma-1}{2}M_n^2}{1+\frac{\gamma-1}{2}M_{n+1}^2}\right)^{\frac{\gamma}{\gamma-1}} \tag{11}$$

$$T_{n+1,j} = T_{n,j}\frac{T_{n+1,j}}{T_{n,j}} = T_{n,j}\left(\frac{1+\frac{\gamma-1}{2}M_n^2}{1+\frac{\gamma-1}{2}M_{n+1}^2}\right) \tag{12}$$

The coefficients of pressure of the respective surface are then easily calculated using the standard equation:

$$C_{p,i,j} = \frac{p_{i,j}-p_\infty}{q_{\infty,j}} \tag{13}$$

For the whole body, the lift and drag coefficients can be calculated using equations (14) and (15), respectively. θ_i is calculated according to equations (5) and (6).

$$C_{L,j} = \frac{1}{2cos(\delta_w)}(-C_{p1,j}cos\theta_{1,j} - C_{p2,j}cos\theta_{2,j} + C_{p3,j}cos\theta_{3,j} + C_{p4,j}cos\theta_{4,j}) \tag{14}$$

$$C_{d,j} = \frac{1}{2cos(\delta_w)}\sum_{i=1}^{4} C_{pi,j}sin\theta_{i,j} \tag{15}$$

where θ_i is the difference in angle between the angle of attack and the local surface inclination. Finally, the pitching moment coefficient about half court is found using equation (16).

$$C_{m,\frac{1}{2}c,j} = \frac{1}{8}(-C_{p1,j}+C_{p2,j}+C_{p3,j}-C_{p4,j}) + \frac{1}{8}tan^2(\delta_w)(C_{p1,j}-C_{p2,j}-C_{p3,j}+C_{p4,j}) \tag{16}$$

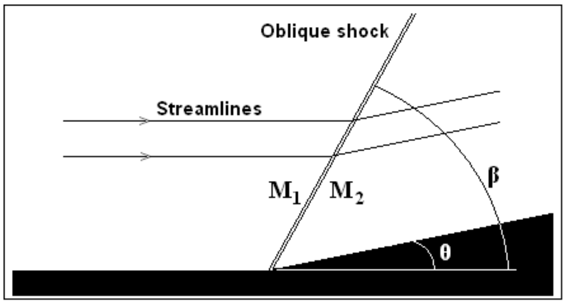
2.2.2 Oblique shock theory

For a given Mach number approaching a point of compression as shows in figure 3(a), the Mach number normal to the shock can be calculated using equation (17).

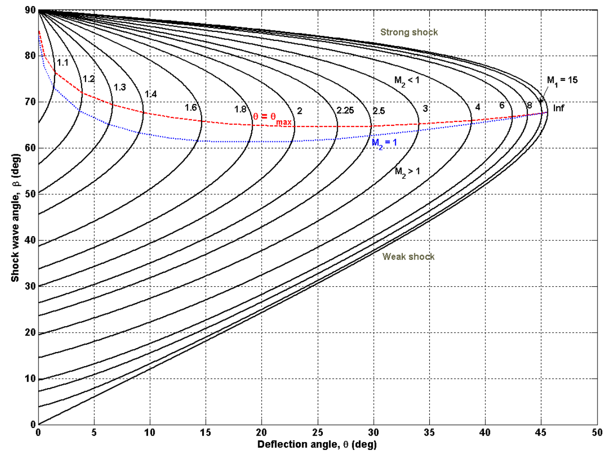
$$M_{k,n,j} = M_{k,j}sin(\beta_j) \tag{17}$$

The oblique shock angle β is a function of the corner angle θ and the Mach number. Either figure 3(b) or equation (18) can be used to find β. Both yield two solutions, however the weak shock is the actual shock occurring in nature since it minimizes total energy. Reference [5] is used to find β. Note that all other values, such as pressure and temperature, are calculated using the presented equations and not the reference.

$$tan\theta_j = 2cot\beta_j\frac{M_j^2sin^2\beta_j - 1}{M_j^2(\gamma+cos(2\beta_j))+2} \tag{18}$$



(a) Oblique shock



(b) $\beta = f(M, \theta)$

Figure 3: Oblique Shock Expansion Theory [4]

The Mach number after the discontinuity is given by equation (19).

$$M_{k+1,j} = \frac{1}{sin(\beta_j - \theta_j)} \sqrt{\frac{1 + \frac{\gamma-1}{2} M_{k,j}^2 sin^2\beta_j}{\gamma M_{k,j}^2 sin^2\beta_j - \frac{(\gamma-1)}{2}}} \tag{19}$$

To calculate the pressure and temperature after the discontinuity, following relationships are used:

$$p_{n+1,j} = p_{n,j} \frac{p_{n+1,j}}{p_{n,j}} = p_{n,j} \left(1 + \frac{2\gamma}{\gamma+1} (M_{k,n,j}^2 - 1) \right) \tag{20}$$

$$T_{n+1,j} = T_{n,j} \frac{p_{n+1,j}}{p_{n,j}} \frac{\rho_{n,j}}{\rho_{n+1,j}} = T_{n,j} \left(1 + \frac{2\gamma}{\gamma+1} (M_{k,n,j}^2 - 1) \right) \left(\frac{(\gamma+1) M_{k,n,j}^2}{2 + (\gamma-1) M_{k,n,j}^2} \right)^{-1} \tag{21}$$

Pressure, lift, wave drag and pitching moment coefficients can the be calculated with equations (13), (14), (15) and (16), respectively.

3 Calculations and Results

The following information was given in order to conduct the analysis. The wing must provide sufficient lift for a 31,500 lbf airplane to fly supersonic speeds at 44,000 ft altitude. The thickness ratio of the airfoil is 0.05 and the wing area is 300 ft^2. Table 2 lists calculated values for the linear theory, table 3 lists the values for shock expansion technique. Finally, table 4 lists calculated pressures and temperatures values found at the respective Mach number for which the wave drag is minimum. Figure 4(a) and 4(b) compare these values.

Table 2: Linear Theory

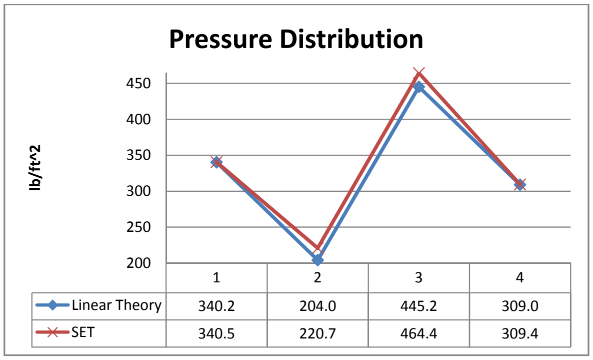
Mach Number	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8
C_L	0.2358	0.1805	0.1426	0.1155	0.0955	0.0802	0.0684	0.0589
α	3.3	3.2	3.1	2.9	2.7	2.5	2.4	2.2
C_d	0.02382	0.01818	0.01429	0.01155	0.00957	0.00809	0.00697	0.00610

Table 3: Shock-Expansion Technique

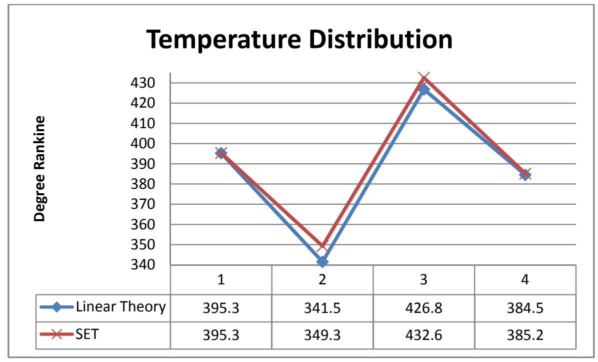
Mach Number	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8
C_L	0.2447	0.1825	0.1436	0.1165	0.0946	0.0806	0.0690	0.0595
C_d	0.02485	0.01845	0.01443	0.01166	0.00958	0.00816	0.00704	0.00615
$C_{m,\frac{1}{2}c}$	0.00946	0.00561	0.00434	0.00367	0.00320	0.00291	0.00267	0.00246

Table 4: Pressure and Temperature Distribution

Method	Pressure	Value	Temperature	Value
Linear Theory M_{∞}=2.8	p_1	340.2	T_1	395.3
	p_2	204.0	T_2	341.5
	p_3	445.2	T_3	426.8
	p_4	309.0	T_4	384.5
SET M_{∞}=2.8	p_1	340.5	T_1	395.3
	p_2	220.7	T_2	349.3
	p_3	464.4	T_3	432.6
	p_4	309.4	T_4	385.2



(a) Pressure distribution



(b) Temperature distribution

Figure 4: Pressure and Temperature distribution at minimum wave drag

3.1 Example Calculations

One set of calculations will be conducted for a Mach number equal to 1.4 for surface 1. Steps are outlined and explained in section two of this report.

3.1.1 Linear Theory

$$q_\infty = \frac{1}{2}\rho_{altitude}\,(1.4\sqrt{\gamma_{air}\,RT_\infty}) = 445.33\,\frac{lb}{ft^2}$$

$$C_L = \frac{31500\,lb}{445.33\,\frac{lb}{ft^2}\,300ft^2} = 0.2358$$

$$\alpha = \frac{0.2358\sqrt{1.4^2-1}}{4} = 0.058\,rad = 3.309\,deg$$

$$\theta_1 = \frac{(0.05\,rad - 0.058\,rad)180\,deg}{\pi} = -0.44\,deg$$

$$C_{P,1} = \frac{2\times 0.44\frac{\pi}{180}}{\sqrt{1.4^2-1}} = -0.0158$$

$$p_1 = -0.0158 * 445.33\frac{lb}{ft^2} + p_{0,altitude} = 317.57\,\frac{lb}{ft^2}$$

$$T_1 = T_{0,altitude}\left(\frac{317.6}{324.6}\right)^{\frac{1.4-1}{1.4}} = 387.55\,R$$

3.1.2 Shock Expansion Technique

$$\nu(M_0) = 8.987$$

$$\nu(M_1) = 8.987 + 0.44 = 9.431$$

which corresponds to a Mach number $M_1 = 1.415$.

$$p_1 = 324.6 \, \frac{lb}{ft^2} \left(\frac{1 + \frac{1.4-1}{2} 1.4^2}{1 + \frac{1.4-1}{2} 1.415^2} \right)^{\frac{1.4}{1.4-1}} = 317.65 \, \frac{lb}{ft^2}$$

$$T_1 = 390 \, R \left(\frac{1 + \frac{1.4-1}{2} 1.4^2}{1 + \frac{1.4-1}{2} 1.415^2} \right) = 387.58 \, R$$

$$C_{p,1} = \frac{317.65 - 324.6}{445.33} = -0.0157$$

4 Discussion of Results

The data acquired from the linear theory is very consistent with the data from the shock-expansion technique. Fluctuations are on the order of 0.001 for the drag and pressure coefficients at Mach 2.8. The pressures and temperatures at the different sections of the airfoil, however, differed up to 19.2 $\frac{lb}{ft^2}$ and 7.8 R. The difference in magnitude can be explained by the assumptions made with each technique. Linear theory assumes small perturbations and, hence, a small angle of deflection (θ). It is a good assumption for angles of deflection of less than 5 degrees, yet some of the angles calculated exceeded this value. As the angles of attack decreased with increasing Mach numbers, θ decreased, and linear theory and shock expansion technique converged, giving very similar values. The drag coefficient decreased with increasing Mach numbers as linear theory predicts by their inverse relationship.

5 Conclusion

The performance of a supersonic wing is analyzed using linear theory and shock-expansion technique. At a given altitude of 44000 ft, a weight of 31500 lb, a thickness ratio of 0.05, and a wing area of 300 ft^2, the lift, wave drag, and pitching moment coefficients of a diamond shaped airfoil are calculated as functions of Mach numbers, ranging from Mach 1.4 to 2.8, and using increments of 0.2. The Mach number at which the wave drag is minimum is found to be 2.8. Pressure and temperature distributions on the surface of the airfoil are plotted at this Mach number. Maximum temperature and pressure are found on section 3 of the airfoil with values of 445.2 $\frac{lb}{ft^2}$ and 464.4 $\frac{lb}{ft^2}$, and 426.8 R and 432.6 R, using linear theory and shock-expansion technique respectively. Minimum temperature and pressure are found on section 2 of the airfoil with values of 204.0 $\frac{lb}{ft^2}$ and 220.7 $\frac{lb}{ft^2}$, and 341.5 R and 349.3 R, using linear theory and shock-expansion technique respectively. The minimum wave drag is found to be 0.00610 and 0.00615, the minimum lift coefficient was found to be 0.0589 and 0.0595, using linear theory and shock-expansion technique respectively. The minimum pitching moment coefficient about half-cord is assumed to be zero using linear theory and is found to be 0.00246 using shock-expansion technique. The data acquired from the linear theory is very consistent with the data from the shock-expansion technique. Fluctuations are on the order of 0.001 for the drag and pressure coefficients at Mach 2.8. The pressures and temperatures at the different sections of the airfoil differ up to 19.2 $\frac{lb}{ft^2}$ and 7.8 R, but can be explained by the assumptions made with each technique.

References

[1] Bertin, John J. (2002). "Aerodynamics for Engineers." 4th edition

[2] Schlichting H. (1979) "Boundary-layer theory." 7th edition, New York: McGraw-Hill Companies, Inc.

[3] "Prandtl-Meyer expansion fan." December 5, 2008 http://en.wikipedia.org/wiki/Prandtl-Meyer_expansion_fan

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[5] "Compressible Aerodynamics Calculator." December 5, 2008 http://www.aoe.vt.edu/~devenpor/aoe3114/calc.html