Experimental Investigation of an Axi-Symmetric Turbulent Jet

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Abstract

A basic understanding of flow characteristics of an axi-symmetric jet is essential to a complete study of Aerodynamics. This experiment was conducted in the California State University of Long Beach wind tunnel to gain a better understanding of these parameters. Profiles of a 1 inch diameter axi-symmetric jet wake were measured at four distances behind the jet. The mean velocity and turbulence profiles were calculated and plotted with a maximum velocity displacement of 1.59 m/s occurring at the centerline a distance back 3.5 times the diameter of the jet. Each profile resembled a somewhat Gaussian profile with a maximum value at its centerline. A linear relationship was found between momentum thickness and distance from the jet exit. A linear relationship was also noticed between the wake thickness and distance from the nozzle exit. These calculations, plots and a discussion of the wake parameters and the effects of its transition downstream of jet exit, including turbulence profiles, are presented within this report.

1 Objective

To determine flow characteristics of an axi-symmetric jet.

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2 Background

A jet is a fluid injected through a nozzle into a stationary fluid. A typical cold jet profile is shown in Figure (1). In these cases, the jets are travelling left to right. The effect of heat release on the structure of inner vortices is investigated by simulating the transitional jet flame without considering the heat release. A depiction of the vortex motion in the cold flame is shown here. Note the vortex merging and loss of coherence of the shear vortices.

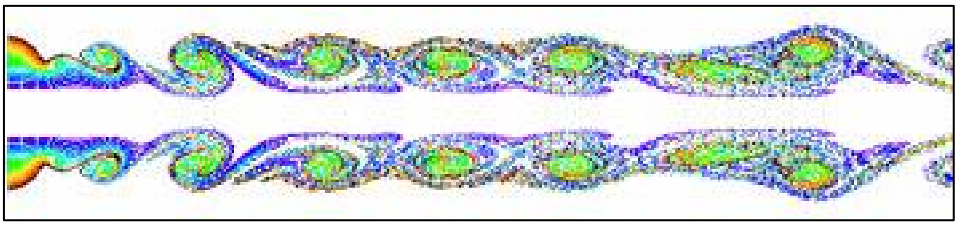

Figure 1: Cold Jet Profile

A typical hot jet profile is shown in Figure (2). A heated-air jet becomes unsteady because of the buoyancy force acting on it. The structure of a 1200-K heated jet is visualized here using a particle-tracking technique. The vortical structures are generated automatically by the UNICORN code in the presence of the buoyancy force in the axial direction. The instantaneous locations of the particles that are injected inside and around a 1200-K air jet are visualized.

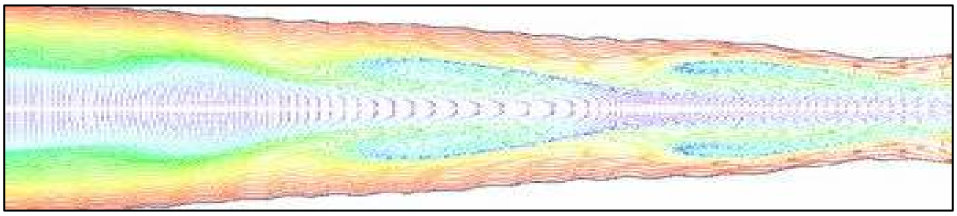

Figure 2: Hot Jet Profile

3 Theory

There is a shear layer formed on the inner surface of the nozzle which grows and exits at the edge of a jet. In between the shear layer is the potential core where the viscosity is zero. The initial velocity profile at the jet outlet is a top hat shape (flat) profile. As the shear layer grows, the potential core disappears and the velocity profile becomes a well known Gaussian profile. For the mean velocity profile, this is the start of self-preservation.

Table 1: Nomenclature

ρ	air density
U_∞	free stream velocity
U_c	mean velocity at the centerline
U_m	maximum free stream velocity
$U_{d\ max}$	defect velocity
u'	fluctuating velocity
D	Jet Diameter
r	radius
θ	wake momentum thickness
ΔP	pressure difference
X	downstream distance
V	Voltage output

4 Procedure

Wind tunnel data was gathered for a cylinder at 4 different free stream velocities. This is the directed procedure that was used:

4.1 Calibration

Calibration of the single hot wire: Turn the digital manometer on and connect it to the hot-wire calibration unit. Set up a single hot wire above the calibration jet and connect it to the constant temperature anemometer and set up the anemometer bridge parameters according to the instruction provided. Turn the compressed air on and record the pressure differential and the output DC voltage for five different speeds.

For calibration data DC output Voltage E in Volts and in H2O were gathered at several different windtunnel airflow speeds. The following relationship is used to determine airstreams velocity:

$$U_{[\frac{m}{s}]} = 19.61 * \sqrt{\Delta P_{[H_2O]}} \tag{1}$$

Calculate the mean velocity from the pressure differential recorded in part one and plot E^2 vs $U^{0.45}$. Find the slope and intercept of the first order polynomial fit to the data. King's Law for isothermal conditions is:

$$E^2 = A + B \times U^N \tag{2}$$

The exponent N is a function of the fluid. For air, this empirical constant is set to be $n = 0.45$.

4.2 Measurements

Connect the compressed air to the central tube of the main combustion unit. Adjust the flow rate to have a mean velocity of 10 m/sec at the center for the jet at the outlet. Place the hot wire at the mid section of the jet at X/D = 3.5. Run the data acquisition program and set the parameters according to the instructions provided. Transverse the hot wire in the vertical direction in steps of 0.1 inch to 4 inch and obtain the statistics across the jet. At each station at least 10 data points should be recorded to obtain the accurate jet profile. Repeat these steps for X/D = 10, 20 and 25.

5 Data

The original data was recorded by a Computer using LABVIEW data acquisition software. Mean values and uncertainties for each point are attached to this report as Attachment No. 1.

6 Calculations

The basic assumption used in all following calculations is that the working fluid, air, is an incompressible fluid. This is a reasonable assumption for low speeds such as those involved in this testing. Standard day atmospheric conditions of air are also used within these calculations. All calculated data is presented within the Tables and Graphs section.

6.1 Velocity Calculations

The maximum mean stream velocity at the center of the jet, U_m, was taken directly from the raw data as the maximum value measured. This was also used as the mean stream velocity at the centerline, U_c, for each distance X from the jet.

$$U_m = U_c$$

To determine the flow profile along the radial direction of the flow, the ratio between mean stream velocity and the maximum mean stream velocity was calculated and plotted against the ratio r/D.

$$\frac{U}{U_m} \; vs \; \frac{r}{D}$$

6.2 Turbulent Velocities

It is also beneficial to understand the magnitude of turbulence within the flow. The velocity sampling root-mean-square, u, from the data acquisition computer can be used as a measure of the turbulent velocities for each reading. The following ratio is used and is plotted against the ratio r/D to give a profile of the turbulence in the radial direction.

$$\frac{\sqrt{u'^2}}{U_m} vs \frac{r}{D}$$

To determine the differences between flow conditions at all four streamline locations the following relationships are useful. These ratios show the relationships of free stream velocity and turbulence magnitudes between each streamline location. These relationships are also plotted.

$$\frac{U_c}{U_0} \; vs \; \frac{X}{D}$$

$$\frac{\sqrt{u'^2}}{U_0}\ vs\ \frac{X}{D}$$

6.3 Momentum Thickness

The momentum thickness for an in-compressible boundary layer is given by equation (3).

$$\theta = \int \frac{u}{u_\infty}(1 - \frac{u}{u_\infty})dy \tag{3}$$

The following formula is used to get a linear approximation of the momentum thickness at each $\frac{X}{d}$ wake location:

$$\theta = \sum \frac{u}{u_\infty}(1 - \frac{u}{u_\infty})\Delta y \tag{4}$$

To show the changes in momentum thickness at different locations downstream of the jet, the following relationship is plotted:

$$\frac{\theta}{D}\ vs\ \frac{X}{D}$$

6.4 Jet Half Width

The jet half width is determined from the mean velocity profiles. To show the relationship between jet half width and distance from the jet nozzle the following can be plotted:

$$\frac{R_{\frac{1}{2}}}{D}\ vs\ \frac{X}{D}$$

6.5 Uncertainty Analysis

In order to get a confidence interval of 95%, we can calculate the error around the mean from our raw data and multiply it by a factor of 2, according to equation (5). For all intervals for each Reynolds number, the maximum of these intervals is chosen to be the confidence interval.

$$CI = 2 \times \delta = 2 \times \sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1}} \tag{5}$$

The CI for the voltages varies between $1.32E-03$ and $8.11E-02$. To simplify calculations, the later value is assumed to be the to be the uncertainty for all four wakes.

Table 2: Calculated Uncertainties

ΔP	$\pm 1.25\ N/m^2$
D	$\pm 0.00005\ m$
Hot Wire	0
y position	$\pm 0.025\ mm$

6.5.1 Uncertainty of ΔP_{pascal} calculations

$$\Delta P_{pascal} = (249) \times \Delta P_{H^20}$$

$$\Delta P_{pascal} = \beta \times \Delta P_{H^20} = \pm 249 * (0005) = \pm 1.254$$

6.5.2 Uncertainty of air speed calculations

$$U_\infty = 1.278 \times \sqrt{\Delta P_{pascal}}$$

$$U_{\sqrt{\Delta P_{psi}}} = \frac{\Delta P_{pascal}}{\sqrt{\Delta P_{pascal}}} \times \frac{\partial(\sqrt{\Delta P_{pascal}})}{\partial \Delta P_{pascal}} \times U_{\sqrt{\Delta P_{pascal}}} = \frac{1}{2} U_{\Delta P_{pascal}} = \frac{1}{2}\sqrt{1} = 0.5$$

$$U_{U_\infty} = \pm \beta \times U_{\Delta P_{pascal}} = \pm 1.278 * 0.5 = \pm 0.3195$$

$$U_{\infty, \frac{m}{s}} = 1.278\ times \sqrt{\Delta P_{pascal}} \pm 0.3$$

6.5.3 Uncertainty of momentum thickness calculations

$$\theta = \int \frac{u}{u_\infty}(1 - \frac{u}{u_\infty})dy$$

$$\theta = f\left(\sum(\frac{u}{u_\infty})\sum(1 - \frac{u}{u_\infty}), \sum \Delta y\right)$$

$$U_\theta = \pm[(U_{U_\infty})^2 + (U_{\Delta y})^2]^{\frac{1}{2}} = \pm[(0.3)^2 + (.0005)^2]^{\frac{1}{2}} = \pm 0.3$$

7 Graphs and Tables

The following graphs display the behavior of the wake and its properties. This table summarizes findings.

Table 3: Summary of Calculations. All units are meter or seconds

Property	Wake 1	Wake 2	Wake 3	Wake 4
U_m	1.59	0.89	0.57	0.48
θ	0.434	0.457	0.792	0.772
U_c	1.59	0.893	0.573	0.482
u'	0.036	0.028	0.0156	0.0165
$\frac{\theta}{D}$	17.1	18.0	31.2	30.0
$r_{\frac{1}{2}}$	0.020	0.122	0.163	0.244
$\frac{r_{\frac{1}{2}}}{d}$	0.8	4.8	6.4	9.6
U_d	0	0.679	1.02	1.11

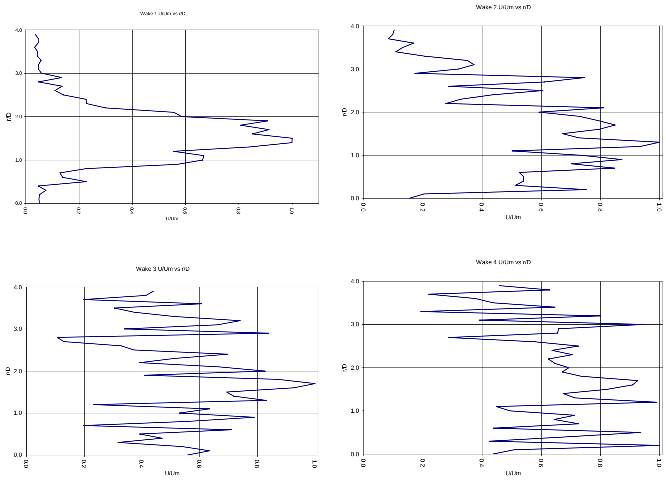

Figure 3: Velocity Profiles

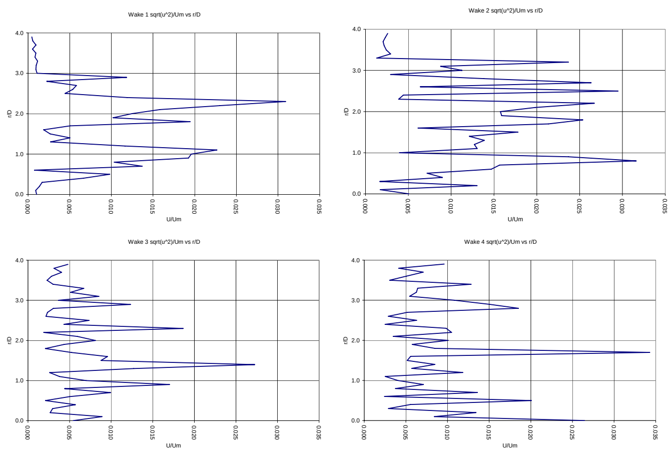

Figure 4: Turbulent Velocity Profiles

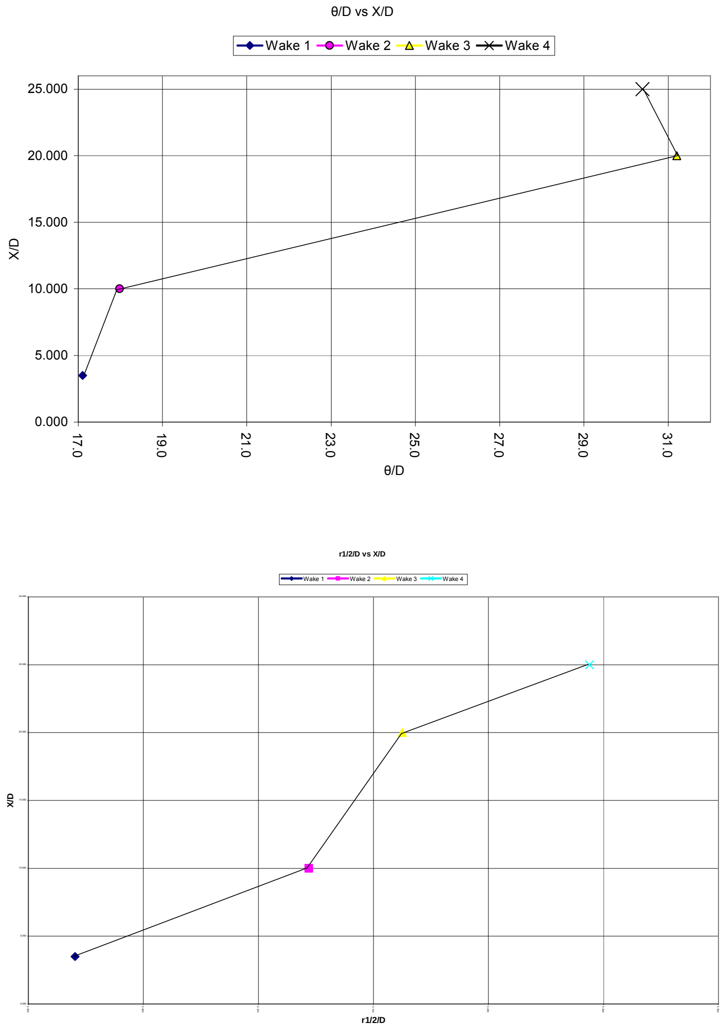

Figure 5: Momentum Thickness and Walk Half Width

8 Discussion of results

The first thing calculated and plotted were the mean stream velocity profiles. As is expected the profiles resemble somewhat a well known Gaussian bell shape curve with the maximum values corresponding to the jet centerline. By these locations, the jet shear layer has grown to the point that the potential core has disappeared. This it not truly the case here. When all four profiles are plotted on the same graph it is easy to see the changes to the mean stream profile as the distance from the jet increases. The width of the wake increases as you go further back from the edge of the jet. The relationship is observed to be linear in the graph of the half wake width and with an error of ($R^2 = .8$) in the graph of the momentum thickness. Trendiness are calculated to be, respectively:

$$\frac{r_{\frac{1}{2}}}{D} = 2.8\frac{X}{D} - 1.6$$

$$\frac{\theta}{D} = 5.31\frac{X}{D} + 10.902$$

The turbulence profiles also resembled a Gaussian curve with the maximum values measured at the jet centerline and similar wake widths as is seen in the free stream velocity profiles. When all four turbulence profiles are plotted on the same graph the differences between the four wake locations from the jet can be shown. The first relationship to notice is that the magnitude of turbulence is at a minimum close to the jet exit increasing downstream, but it falls back down.

The fluctuating velocities were plotted and graphed. This relationship shows high fluctuations and a decrease in centerline strength can be followed through all 4 plots. The centerline velocity slowly tapers off back to zero. The plot of shows a similar curve.

9 Conclusions and recommendations

Mean stream velocity and turbulence profiles were measured at four locations downstream of an axi-symmetric jet. Through the evaluation and analysis of the data gathered a better understanding of the flow characteristics of an axi-symmetric jet was accomplished.

Both the mean stream velocity and turbulence profiles resemble somehow Gaussian curves with maximum values at the centerline. This suggests that the jet shear layer has not yet fully grown to the point that the potential core disappears. The width of the wake follows a linear relationship with the distance from the jet exit as does the momentum thickness. The centerline velocity and turbulence values drop off, or exponentially, from the maximum at the jet exit. These values approach zero as the distance from the jet exit increases.

It would have been of interest to gather additional data further away from the jet. Data is still very fluctuation and some relationships are hard to see.

References

[1] Dr. Hamid Rahai, MAE 440 Aerodynamics Laboratory Experiments, California State University Long Beach, Spring 2007

[2] John J. Bertin, Aerodynamics for Engineers, 4th edition, 2002