Turbulent Wake of a Smooth Cylinder

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Abstract

A basic understanding of flow characteristics past a circular cylinder is essential to a complete study of Aerodynamics. This experiment was conducted in the California State University of Long Beach wind tunnel to gain a better understanding of the wake parameters and characteristics behind a circular cylinder. Profiles of the disturbed wake were measured at four distances behind a cylinder at a flow speed of 3.5 m/s. The mean velocity profiles were calculated and plotted with a maximum velocity displacement of 0.83 m/s occurring at a distance back 42 times the diameter of the cylinder. Momentum thickness values were also calculated using this data and were used to determine a Coefficient of drag value of 0.807 for this cylinder. These calculations and a discussion of the wake parameters and the effects of its transition downstream of the cylinder, including turbulence profiles, are presented within this report.

1 Objective

To become familiar with a circular cylinder wake and its parameters.

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2 Background and Theory

Wake flow of a cylinder with arbitrary cross section orthogonal to a uniform flow is characterized by a moment defect associated with the drag of the cylinder. Figure (1) depicts flow around a stationary cylinder.

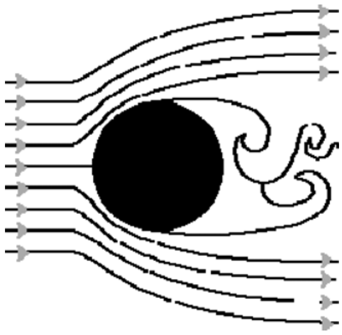

Figure 1: Cylinder with Viscous Cross Flow

The drag is a total drag that includes both viscous and form (pressure) drag. In the far downstream of the wake the streamlines are nearly uniform and we can assume a uniform pressure everywhere. At this location the momentum thickness is constant and the drag coefficient of the cylinder can be found from the following equation:

$$C_d = \frac{F_D}{\frac{1}{2}\rho U_\infty^2} = \frac{2\theta}{d} \tag{1}$$

Variables are defined in Table (1). In the near field of the cylinder, the pressure is not uniform and there exists an axial pressure gradient which causes reduction in the momentum thickness. Thus it is important to identify the location where the axial pressure gradient is zero, before the momentum thickness is used to calculate the drag coefficient. Additional turbulent wake parameters of importance include the following: Wake half-width, $Y_{\frac{1}{2}}$, the vertical location where the mean deflect velocity is half. Axial variation of the mean and turbulent velocities U_c and $\sqrt{u'^2}$ along the center line. Maximum mean defect velocity,$U_{max} = U_\infty - U_e$.

3 Procedure

Experiment 4 was conducted at California State University of Long Beach with the Lab Wind Tunnel. The following procedures were used. Prior to conducting the experiment the pitot tube was calibrated inside the wind

Table 1: Nomenclature

C_d	drag coefficient
F_D	drag force
ρ	air density
U_∞	free stream velocity
$U_{d\ max}$	maximum mean defect velocity
U_c	mean velocity at the centerline
U_e	mean velocity at the edge of the wake
$u^{'}$	fluctuating velocity
d	diameter of object
θ	wake momentum thickness
$Y_{\frac{1}{2}}$	wake half-width
ΔP	pressure difference

tunnel using the digital manometer. Using different wind speeds the lab computer was calibrated. A $\frac{1}{2}$ in diameter, d, cylinder was placed inside the wind tunnel as shown in Figure (2) by the lab instructor with the hotwire placed a distance $x = 32D$ downstream from the edge of the cylinder. The free stream velocity was set to 3.5 m/s. The wake profile was recorded using the data acquisition program with roughly 400 data points gathered. The spacing between data points was more dense in the center of the wake. This procedures was repeated at locations x = 42D, 66D and 90D.

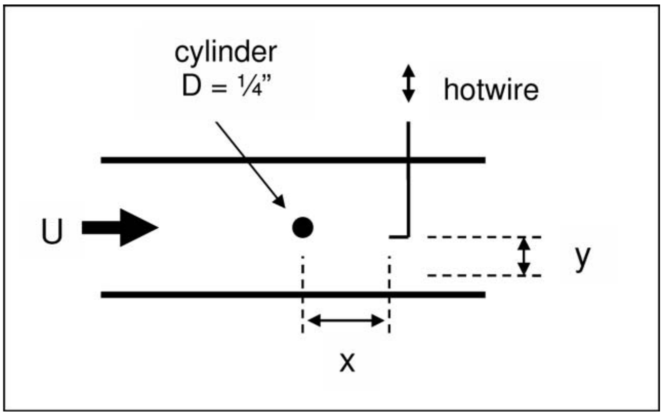

Figure 2: Windtunnel Setup

4 Data

The original data was recorded by a Computer using LABVIEW data acquisition software. Mean values and uncertainties for each point are attached to this report as Attachment No. 1.

5 Calculations

The basic assumption used in all following calculations is that the working fluid, air, is an incompressible fluid. This is a reasonable assumption for low speeds such as those involved in this testing. Standard day atmospheric conditions of air are also used within these calculations. All calculated data is presented within the Tables and Graphs section.

5.1 Hot Wire Calibration

For calibration data DC output Voltage E in Volts and in H2O were gathered at several different windtunnel airflow speeds. The following relationship is used to determine airstreams velocity:

$$U_{[\frac{m}{s}]} = 19.61 * \sqrt{\Delta P_{[H_2O]}} \tag{2}$$

By plotting the relationship E^2 vs. $U^{0.45}$ at the points recorded, the corresponding slope of the fitted polynomial and the intercept gives us constants B and A, respectively, of King's Law. The exponent N is a function of the fluid. For air, this empirical constant is set to be $n = 0.45$.

$$E^2 = A + B \times U^N \tag{3}$$

5.2 Velocity Calculations

The local mean stream velocity at the edge of the wake, U_e , was given in the raw data of the computer. For uniform free stream mean velocity, this value is equal to the free stream mean velocity U_∞. With the mean stream velocity at the edge of the wake known it is possible to find the velocity profiles for each set of data. Plots of these profiles vs. the ratio y/d are presented within the Tables and Graphs section. It is now also possible to find the mean defect velocity. This is the difference between the mean velocity at the centerline and the freestream velocity.

$$U_{dmax} = U_\infty - U_c \tag{4}$$

5.3 Turbulent Velocities

The root-mean-square, u, of the velocity sampling from the data acquisition computer can be used as a measure of the turbulent velocities for each reading. The following ratio is used and is plotted against the ratio y/d to give a profile of the turbulence in the y direction.

$$\frac{\sqrt{U^2}}{U_e} \tag{5}$$

5.4 Momentum Thickness

The momentum thickness for an in-compressible boundary layer is given by equation (6).

$$\theta = \int \frac{u}{u_\infty}(1 - \frac{u}{u_\infty})dy \tag{6}$$

The following formula is used to get a linear approximation of the momentum thickness at each $\frac{X}{d}$ wake location:

$$\theta = \sum \frac{u}{u_\infty}(1 - \frac{u}{u_\infty})\Delta y \tag{7}$$

5.5 Coefficient of Drag

The coefficient of drag is calculated using equation (1).

5.6 Uncertainty Analysis

In order to get a confidence interval of 95%, we can calculate the error around the mean from our raw data and multiply it by a factor of 2, according to equation (8). For all intervals for each Reynolds number, the maximum of these intervals is chosen to be the confidence interval.

$$CI = 2 \times \delta = 2 \times \sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1}} \tag{8}$$

The CI for the voltages varies between $1.32E-03$ and $8.11E-02$. To simplify calculations, the later value is assumed to be the to be the uncertainty for all four wakes.

Table 2: Calculated Uncertainties

ΔP	$\pm 1.25\ N/m^2$
D	$\pm 0.00005\ m$
Hot Wire	0
y position	$\pm 0.025\ mm$

5.6.1 Uncertainty of ΔP_{pascal} calculations

$$\Delta P_{pascal} = (249) \times \Delta P_{H^20}$$

$$\Delta P_{pascal} = \beta \times \Delta P_{H^20} = \pm 249 * (0005) = \pm 1.254$$

5.6.2 Uncertainty of air speed calculations

$$U_\infty = 1.278 \times \sqrt{\Delta P_{pascal}}$$

$$U_{\sqrt{\Delta P_{psi}}} = \frac{\Delta P_{pascal}}{\sqrt{\Delta P_{pascal}}} \times \frac{\partial(\sqrt{\Delta P_{pascal}})}{\partial \Delta P_{pascal}} \times U_{\sqrt{\Delta P_{pascal}}} = \frac{1}{2} U_{\Delta P_{pascal}} = \frac{1}{2}\sqrt{1} = 0.5$$

$$U_{U_\infty} = \pm\beta \times U_{\Delta P_{pascal}} = \pm 1.278 * 0.5 = \pm 0.3195$$

$$U_{\infty, \frac{m}{s}} = 1.278\ times \sqrt{\Delta P_{pascal}} \pm 0.3$$

5.6.3 Uncertainty of momentum thickness calculations

$$\theta = \int \frac{u}{u_\infty}(1 - \frac{u}{u_\infty})dy$$

$$\theta = f\left(\sum(\frac{u}{u_\infty})\sum(1 - \frac{u}{u_\infty}), \sum \Delta y\right)$$

$$U_\theta = \pm[(U_{U_\infty})^2 + (U_{\Delta y})^2]^{\frac{1}{2}} = \pm[(0.3)^2 + (.0005)^2]^{\frac{1}{2}} = \pm 0.3$$

5.6.4 Uncertainty of coefficient of drag calculation

$$C_d = \frac{F_D}{\frac{1}{2}\rho U_\infty^2} = \frac{2\theta}{d}$$

$$C_d = f(\theta, d)$$

$$U_{C_d} = \pm\left[(\frac{\theta}{C_d}\frac{\partial C_d}{\partial \theta})^2 + (\frac{d}{C_d}\frac{\partial C_d}{\partial d})^2\right]^{\frac{1}{2}} = [U_\theta^2 + U_d^2]^{\frac{1}{2}} = \pm 0.3$$

$$C_d = \frac{2\theta}{d} \pm 0.3$$

6 Graphs and Tables

The following graphs display the behavior of the wake and its properties. Note that the first value is before we have constant momentum thickness. Therefore, the first value might not follow the expected trend. The following table summeriness calculated values:

Table 3: Summary of Calculations

Data Set	$\frac{X}{d}$	U_∞	U_c	$U_{d,max}$	$\frac{U_{d,max}}{d}$	θ	$\frac{\theta}{d}$	$Y_{\frac{1}{2}}$	$\frac{Y_{\frac{1}{2}}}{d}$	C_d
		$[\frac{m}{s}]$	$[\frac{m}{s}]$	$[\frac{m}{s}]$		$[m]$		$[in]$		
1	32	3.39	2.75	0.64	50.50	3.505E-03	0.276	1.6	3.2	0.552
2	42	3.42	2.60	0.83	64.99	5.072E-03	0.399	1.4	2.8	0.799
3	66	3.46	2.78	0.68	53.94	4.046E-03	0.319	1.3	2.6	0.637
4	90	3.58	3.00	0.58	45.63	6.261E-03	0.493	1.1	2.2	0.986

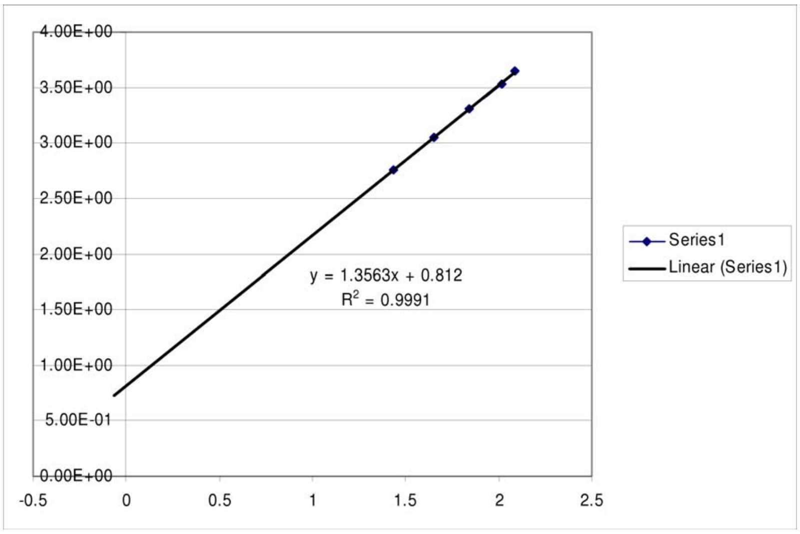

Figure 3: Calibration

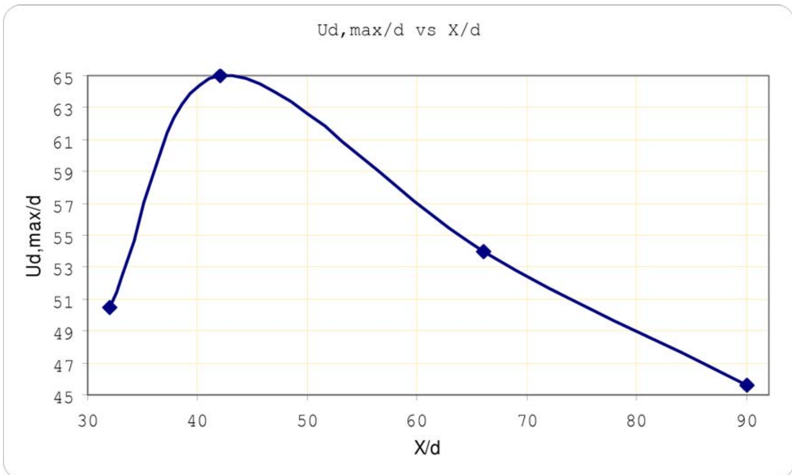

Figure 4: Maximum Displacement Velocity

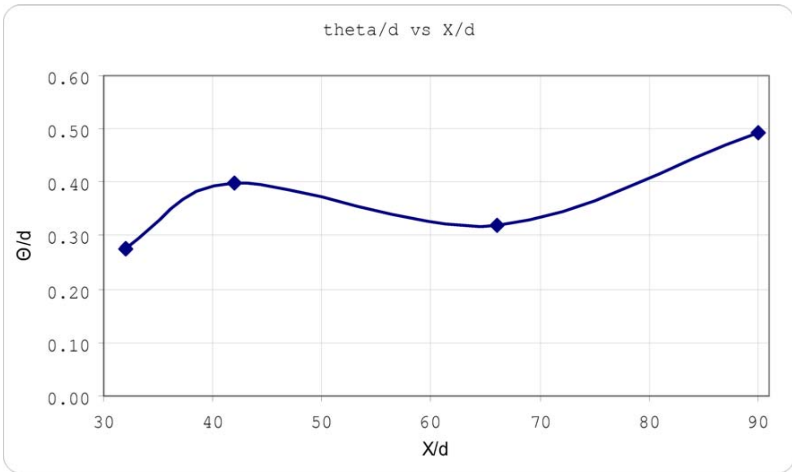

Figure 5: Momentum Thickness Development

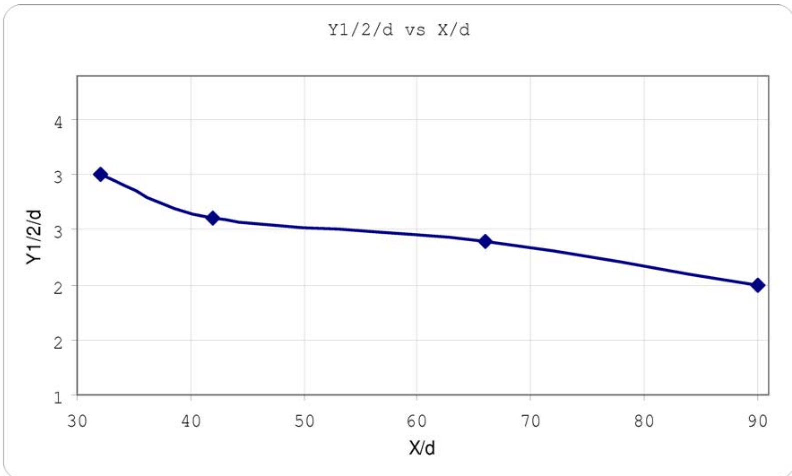

Figure 6: Halfwidth of the Wake

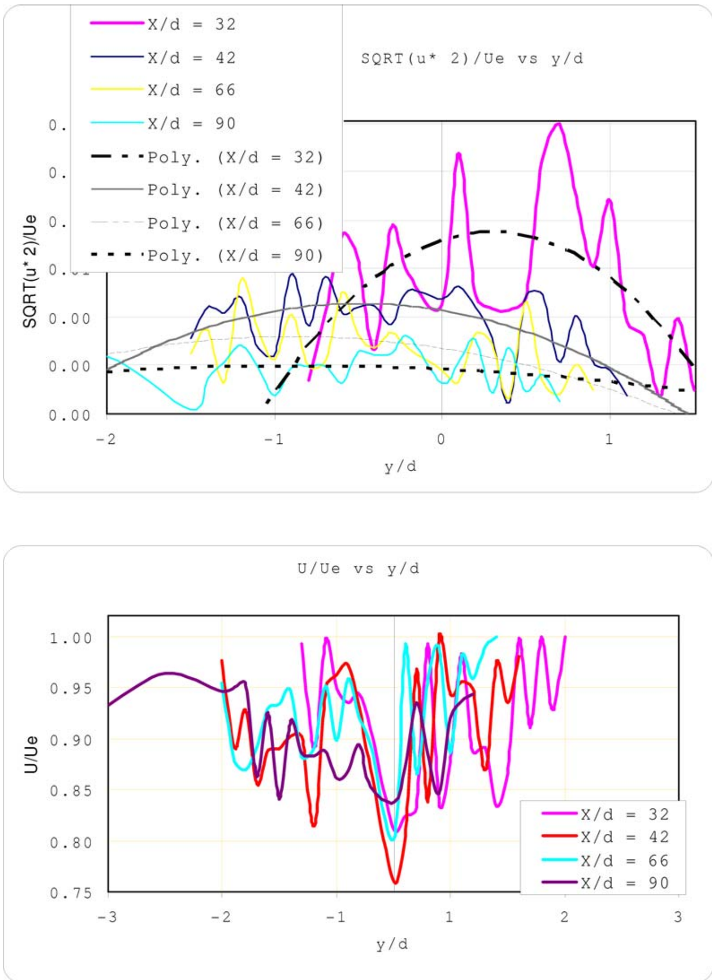

Figure 7: Velocity Comparison

7 Discussion of results

The first thing calculated and plotted were the mean stream velocity profiles. As is expected the stream velocity experiences the largest drop in value directly at the center line of the cylinder. The velocity profile then follows a non smooth, oscillating line, back until it reaches the edge of the wake and is equal to the free stream or edge velocity.

What was interesting to find was that the maximum displacement velocity did not occur at X/d = 32. It actually occurred at the next data set which was further back from the cylinder at X/d = 42. This relationship also shown when maximum velocity displacement vs X/d is plotted. Since the wake should decreases in size as it moves downstream and X/d = 32 is at a location where the flow is not yet fully developed, these values are expected.

Comparing the Velocity ratios, Part two of Figure (7) shows that the ratio is minimum at X/d = 42 and the decreases downstream. These values go hand in hand with the displacement velocities. Velocities at the centerline of the Wake ranged from $2.6\frac{m}{s}$ to $3.0\frac{m}{s}$, excluding the first data set.

The momentum thickness varies from 3.5 mm at the first location to 6.3 mm at the final location. It still seems to be slowly increasing. Since fully developed flow is expected at X/d = 40, the three value should be quiet constant, but they vary. However, since lots of data seems not to be very smooth, some disturbance behavior can vary these results.

Also shown in these graphs is the wake size and how it decreases with distance away from the cylinder. This is also graphed as the wake half-width vs. X/d. The relationship is non-linear suggesting that there is a limit to the size of the downstream wake. Since the wake is not fully developed at the first measurement, data might not be comparable with the later three sets.

By plotting the turbulent flow profile, one is able to see the wake size as well as how the turbulent effects the cylinder has on the air stream. In the first profile, at X/d = 30, the magnitude of turbulence is at a maximum and is constant over the thickness of the cylinder. This is true for the three other profiles as well, however by moving back further along the horizontal, the width of the turbulent flow peak grew slightly wider. Turbulent flow transitions at these peaks. The magnitude of peak turbulence also decreased with the distance back from the cylinder. This relationship is easy to see in the figure with all four turbulent profiles plotted on the same graph. Furthermore, it seems that the flow is not symmetric. The centerline shifts downwards downstream.

The Coefficient of drag was calculated at each horizontal position. Since the position at X/d = 32 was before the location of the axial pressure gradient equal to zero, this value was not reliable and therefore not considered. The average of the other three Coefficient of drag values, 0.807315, was within the expected range of about 0.8.

8 Conclusions and recommendations

Wind tunnel testing was conducted on a flat plate to gain a better understanding of the wake downstream of a cylinder. A freestream velocity of 3.5 m/s was used and the mean stream velocity profile was measured using a hotwire at X/d distances from the edge of the cylinder of 32 to 90.

Using the data obtained the mean velocity and turbulent flow profiles were graphed at each location. These graphs matched expected profiles, however they are not smooth as they should have been. The wake thicknesses at each location were determined graphically and momentum thickness values were calculated using the data. These values were all within expected ranges. Using the moment thickness, the Coefficient of drag was determined to be 0.81 which is a realistic value.

The maximum velocity displacement of 0.83 m/s occurred at a distance X/d = 42 from the edge of the cylinder. It is believed that the flow still transitioned at this location, maybe due to disturbances.

It also seems that the freestream velocity is far below the expected experimental value. This could be the reason that our data was not smooth since we were in a different Reynolds Number flow environment and lacked momentum. For this experiment, the calculated Reynolds number turned out to be around 2500, way below any turbulent flows! However, relative values still displayed the relationship and development of the wake downstream.

References

[1] Dr. Hamid Rahai, MAE 440 Aerodynamics Laboratory Experiments, California State University Long Beach, Spring 2007

[2] John J. Bertin, Aerodynamics for Engineers, 4th edition, 2002