Measurement of Boundary Layer on a Flat Plate

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Abstract

A basic understanding of flow characteristics over a flat plate is essential to a complete study of Aerodynamics. This experiment was conducted in the California State University of Long Beach, $CSULB$, windtunnel to gain a better understanding of the parameters and characteristics of fluid flow over a flat plate. Readings of the boundary layer were taken at four locations along a flat plate at an average free stream velocity U_∞ of $19.1 \pm 0.3 \, \frac{m}{s}$ giving Reynolds numbers corresponding to laminar through turbulent flows. The height of the boundary layer ranged from around 3 mm to 29 mm. Displacement thickness and momentum thickness values were calculated using the velocity profile. The skin-friction coefficients were determined using three separate techniques all leading to similar, yet different results. Comparing these results to a theoretical value of 0.0037, the best result for C_f was calculated to be 0.00372 using an equation in terms of Reynolds number for a turbulent section.

1 Objective

To become familiar with a boundary layer and its parameters.

2 Background and Theory

Boundary layer is a layer adjacent to a surface where viscous effects are important. Figure (1) depicts flow of a fluid over a flat plate.

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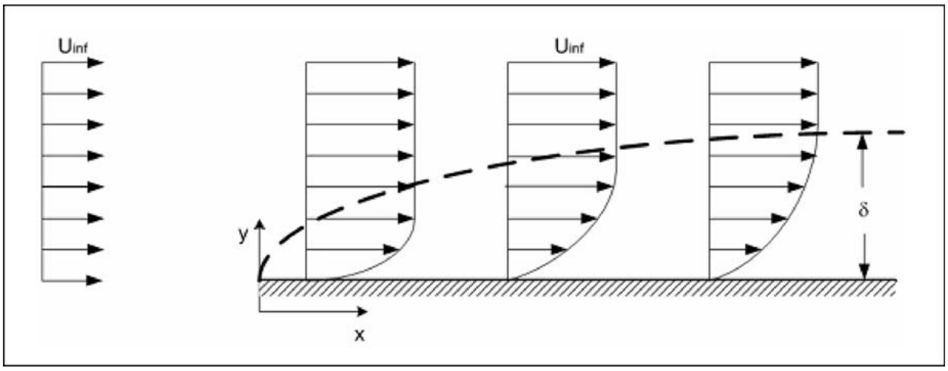

Figure 1: Flow over a flat plate

The fluid particles at the flat plate surface have zero velocity and they act as a retardant to reduce velocity of adjacent particles in the vertical direction. Similar actions continue by other particles until at the edge of the boundary layer where the particles' velocity is 99% of the free stream velocity. Boundary layers can also be measured by more significant parameters. The main boundary layer parameters are as follows: The displacements thickness, δ^* is defined as the distance by which the external streamlines are shifted due to the presence of the boundary layer:

$$\delta^* = \int (1 - \frac{u}{u_\infty}) dy \tag{1}$$

The momentum thickness represents the height of the free-stream flow which would be needed to make up the deficiency in momentum flux within the boundary layer due to the shear force at the surface. The momentum thickness for an in-compressible boundary layer is given by:

$$\theta = \int \frac{u}{u_\infty} (1 - \frac{u}{u_\infty}) dy \tag{2}$$

The skin-friction coefficient is defined as:

$$C_f = \frac{\tau_0}{\frac{1}{2}\rho u_\infty^2} dy \tag{3}$$

$$\tau_0 = (\frac{\partial u}{\partial y})_{y=0} \tag{4}$$

The Reynolds number is a measure of the ratio of inertia forces to viscous forces. It can be used to characterize flow characteristics aver a flat plate. Values under 500,000 are classified as Laminar flow where values from 500,000 to 1,000,000 are deemed Turbulent flow. Is it important to distinguish between turbulent and non turbulent flow since the boundary layer thickness varies, as Fig. (2) shows.

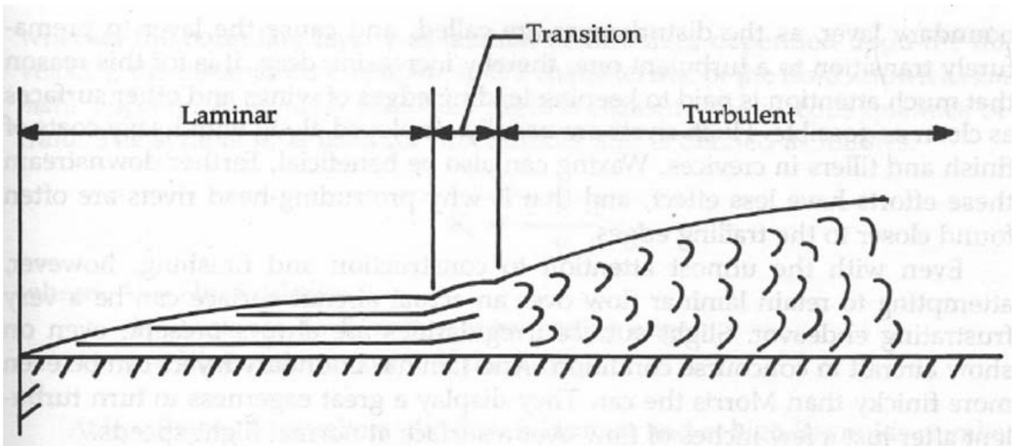

Figure 2: Flow over a flat plate

3 Procedure

Experiment 3 was conducted at California State University of Long Beach with the Lab Wind Tunnel. The following procedures were used. The wind tunnel was setup by the lab instructor with a pitot tube placed 12" from the leading edge of a flat plate. The wind tunnel was turned on and the digital manometer was calibrated. The pressure differential ΔP at at least 26 points was measured within the boundary layer with a Δy of 0.05 inches. At each interval the mean pressure differential was averaged over 10 data points and recorded on the PC. The pitot tube was adjusted to 24", 36" and 48" and above steps were repeated.

4 Data

The original data was recorded by a Computer using LABVIEW data acquisition software. Mean values and uncertainties for each point are attached to this report as Attachment No. 1.

5 Calculations

The basic assumption used in all following calculations is that the working fluid, air, is an incompressible fluid. This is a reasonable assumption for low speeds such as those involved in this testing. Standard day atmospheric conditions of air are also used within these calculations. All calculated data is presented within the Tables and Graphs section.

Table 1: Nomenclature, SLS Conditions

C_d	drag coefficient
F_D	drag force
ρ	air density
U_∞	free stream velocity
μ	dynamic viscosity
ν	kinematic viscosity
P_∞	free stream pressure
P_0	stagnation pressure
ΔP	pressure difference
L	Length of object

5.1 Effective Center

The effective center equation is used to measure the first Δy distance on which data is taken at each location. This is a function of the outer and inner diameter of the Pitot tube. Measured values are $D = 0.05$" and $d_i = 0.025$".

$$y_{ec} = (0.131 + 0.82\frac{d_i}{D}) * D = 0.69mm \tag{5}$$

5.2 Free Stream Velocity

The recorded data for the experiment included Pressure readings with the units of in-H^2O. This data had to be converted into Pascal's for velocity calculations. Equations (6) and (7) were used for conversion and free stream velocity calculations.

$$\Delta P_{Pascal} = 249 \times \Delta P_{H^2O} \tag{6}$$

$$U_\infty = \sqrt{\frac{\Delta P_{Pascal}}{\frac{1}{2}\rho_{Air,SL}}} = 1.278\sqrt{\Delta P_{Pascal}} \tag{7}$$

Applying Equation (7), the free stream velocities for the conducted experiments ranged between $18.7 \pm .3\,\frac{m}{s}$ and $19.6 \pm .3\,\frac{m}{s}$.

5.3 Reynolds Number

Having found the free stream velocity earlier it is possible to calculate the Reynolds number for all four flow conditions using the following relationship:

$$Re_\infty = \frac{\rho U_\infty L}{\mu} = \frac{U_\infty L}{\nu} \tag{8}$$

The length L was measured from the leading edge of the flat plate at which the boundary layer distributions are being evaluated were measured in inches and were converted to meters.

Table 2: Reynolds numbers and flow types as a function of L

Data Set	Length L in meter	Reynolds Number	Flow Type
1	0.3048	3.4×10^5	Laminar
2	0.6096	6.9×10^5	Transition
3	0.9144	1.0×10^6	Turbulent
4	1.2192	1.3×10^6	Turbulent

5.4 Displacement Thickness

Once the free stream velocity and velocities at each Δy interval are known, the displacement thickness δ^* can be calculated according to equation (1). The following formula is used to get a linear approximation of the displacement thickness at all four pitot tube locations.

$$\delta^* = \sum(1 - \frac{u}{u_\infty})\Delta y \tag{9}$$

The thickness of the boundary layer itself is a function of Reynolds number. The boundary curve for turbulent flow is much steeper. These are the equations used to calculate δ for laminar and turbulent flow, respectively.

$$\delta_L = \frac{5 \times L}{Re_L^{.5}}$$

$$\delta_T = \frac{.382 \times L}{Re_L^{.2}}$$

5.5 Momentum Thickness

The momentum thickness for an in-compressible boundary layer is given by equation (2). The following formula is used to get a linear approximation of the momentum thickness at all four pitot tube locations.

$$\theta = \sum \frac{u}{u_\infty}(1 - \frac{u}{u_\infty})\Delta y \tag{10}$$

With displacement and momentum thickness found, H can be calculated:

$$H = \frac{\delta^*}{\theta}$$

5.6 Skin friction Coefficient

The skin-friction coefficient can be evaluated using a variety of techniques:

5.6.1 Clauser Chart

By evaluating the Clauser chart, the skin-friction coefficients can be found. For all four locations the lowest values were taken for use in the Clauser chart, corresponding to equation (5). The corresponding skin-friction values C_f were read from the Clauser Chart.

5.6.2 Reynolds Number

The skin-friction coefficients can be calculated using Reynolds number with these equations a laminar or turbulent boundary layer, respectively.

$$C_{f,L} = \frac{0.664}{\sqrt{Re}}$$

$$C_{f,T} = \frac{0.0583}{Re^{0.2}}$$

5.6.3 Momentum Thickness

Another way to calculate the skin friction coefficient is to calculate the slope of θ vs the length L. With a zero pressure gradient,

$$u\frac{du}{dx} = -\frac{1}{\rho}\frac{dP}{dx} = 0$$

the Von Karman integral equation

$$\frac{d\theta}{dx} + (2\theta + \delta^*)\frac{1}{u_\infty}\frac{du_\infty}{dx} = \frac{\tau_0}{\rho u_\infty^2}$$

becomes, using the relationship of equation (3)

$$C_f = 2\frac{d\theta}{dx} \tag{11}$$

Fig. (4) shows the approximated, linear θ value using this method.

Table 3: Summary of Skin Friction Coefficients, 10^{-3}

Data Set	C_f Clauser Chart	C_f Momentum Thickness	C_f Reynolds Number
1	6.0	2.4	1.10
2	6.0	2.4	4.02
3	4.5	2.4	3.72
4	4.5	2.4	3.54

5.7 Uncertainty Analysis

In order to get a confidence interval of 95%, we can calculate the error around the mean from our raw data and multiply it by a factor of 2, according to equation (12). For all intervals for each Reynolds number, the maximum of these intervals is chosen to be the confidence interval.

$$CI = 2 \times \delta = 2 \times \sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1}} \tag{12}$$

The CI for ΔP varies between 6.42 Pascal and 7.18 Pascal. To simplify calculations, the value of 7.18 Pascals is used for all uncertainty calculations. The ΔP measured uncertainty is 1.25 Pascal.

Table 4: Calculated Uncertainties

ΔP	$\pm 1.25\ N/m^2$
D	$\pm 0.00005\ m$
U_∞	$\pm 0.3\ m/s$
θ	± 0.3
Re	$\pm.03$

5.7.1 Sample Calculation

$$\delta^* = f\left(\sum(1 - \frac{u}{u_\infty}), \sum \Delta y\right)$$

$$U_{\delta^*} = \pm[(U_{U_\infty})^2 + (U_{\Delta y})^2]^{\frac{1}{2}} = \pm[(0.00246)^2 + (.05 * 2.54/100)^2]^{\frac{1}{2}} = 0.0028$$

6 Graphs and Tables

Calculated Pressure differentials, velocity distributions, displacement and momentum thicknesses, and skin friction coefficients are attached to this report as Attachment No. 2.1 to 2.4.

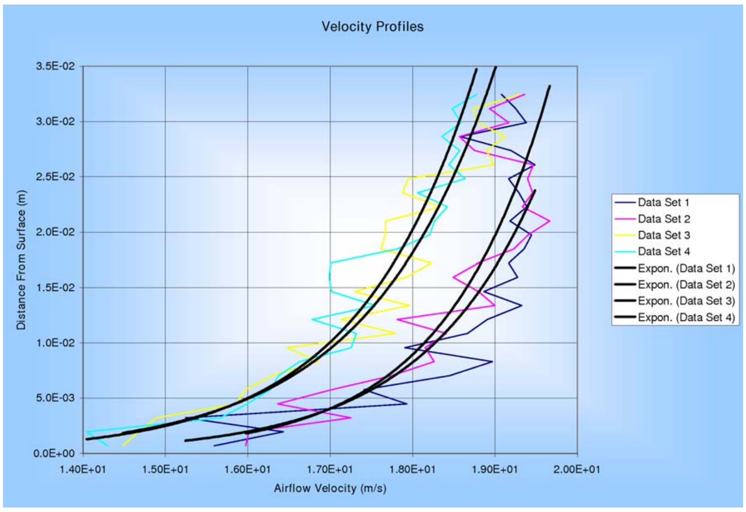

Figure 3: Velocity profile

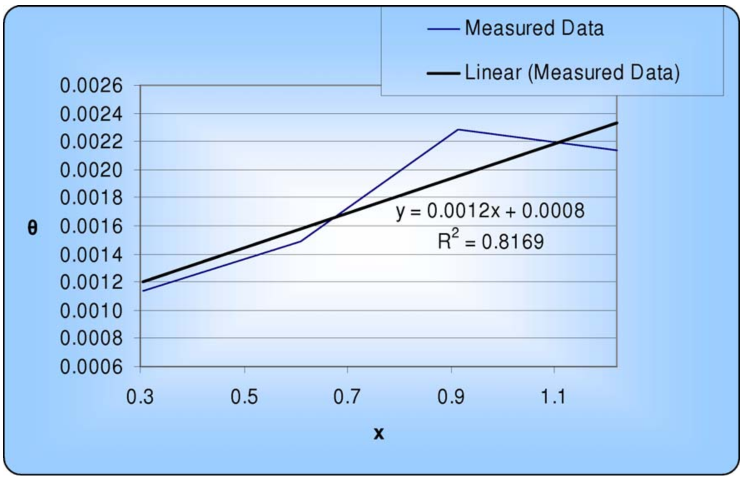

Figure 4: θ vs. Length

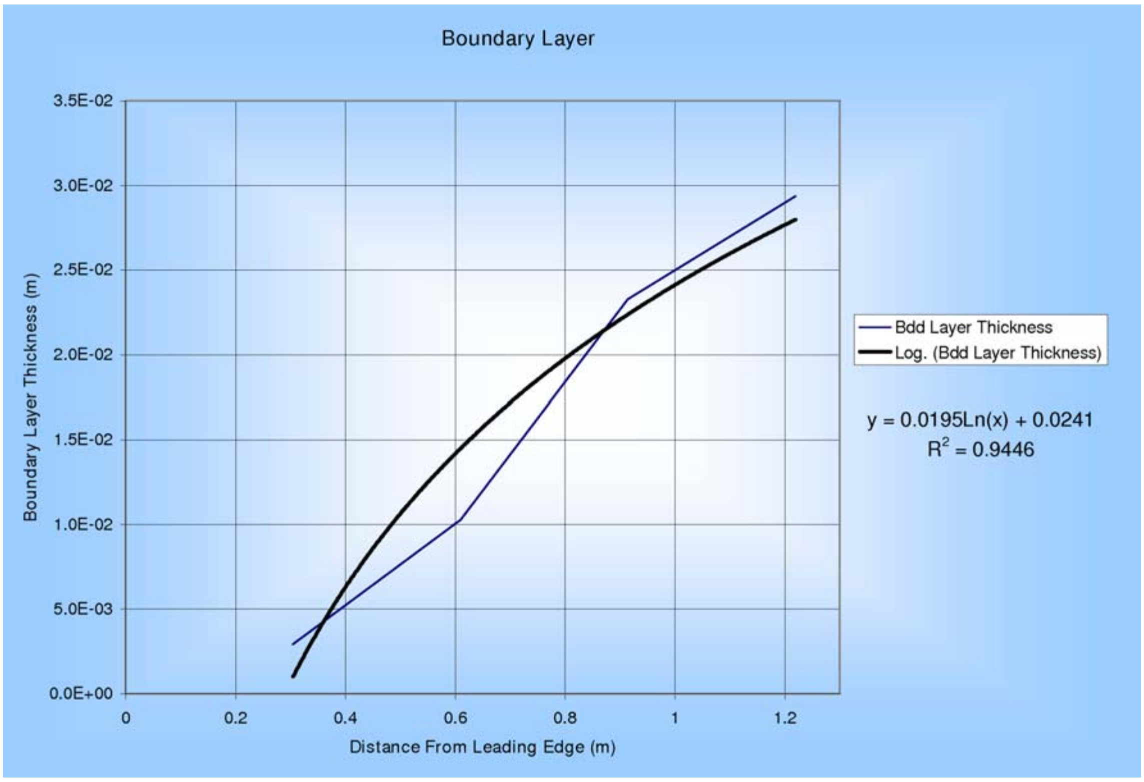

Figure 5: Thickness of the Boundary Layer

7 Discussion of results

The Reynolds numbers for the flow are within the sub-critical Reynolds number regime. The flow transitioned from a laminar to a turbulent flow (Re = 500,000) prior to the second location, L = 24”. The boundary layer thicknesses were in the expected ranges with respect to the L location along the flat plate. The data shows the thickness increasing along the length of the flat plate. Figure (5) shows the data and a logarithmic interpolation. The results indicate that the behavior of a boundary layer is largely a function of the Reynolds number. The Reynolds number is a function of the flow speed, viscosity and density of the fluid. Separation occurs earlier and with more strength for higher Reynolds numbers. It is also useful to note that the shape and the characteristic length of the surface make a big difference in the boundary layer parameters.

The mean velocity graphs from Fig. (3) visually show the velocity distribution within the boundary layer thickness. The graph shows that the boundary layer grows as L is increased and the curves tend to have a greater tangent as velocity increases. Also, the calculated displacement thickness and momentum thickness values were also in the expected ranges. The theoretical values compared to the calculated values of skin-friction coefficient did not match up at all points. It could be concluded that at this location, x = 24", the flow was in fact, still laminar or possibly in transition. This would help explain the differences in theoretical vs. calculated skin-friction coefficients. It may also be a fair assumption since the Reynolds number at this location was just barely over the transition value. Due to this assumption, the calculated boundary later thickness is assumed to be the mean of the respective laminar and turbulent calculated value.

8 Conclusions and recommendations

Windtunnel testing was conducted on a flat plate to gain a better understanding of boundary layers and there parameters. Readings of the boundary layer were taken at four locations along the flat plate at an average flow speed calculated to be $19.1 \pm 0.3\,\frac{m}{s}$ giving Reynolds numbers in the range of 341,000 to 1,300,000. These values correspond to laminar through turbulent flows and are within the sub-critical Reynolds number regime. Using the data obtained the mean velocity profiles were graphed at each location. These graphs matched expected profiles. The boundary layer thicknesses at each location were determined and displacement thickness and momentum thickness values were calculated using the data. The skin-friction coefficients were determined using three separate techniques. These values were not unreasonably away from each other, however they did vary. The discrepancy between the skin-friction coefficient of the second reading would lead us to re-evaluate wether the flow at this location was truly turbulent. The calculated results suggest that this location was in fact still laminar or possibly in transition. It might be useful not to use an open windtunnel for this experiment since small disturbances can lead to fatal uncertainties, since the desired calculated data is very sensible and small in magnitude. That might explain some of the errors. Furthermore, it would be useful to take more data within the laminar flow range to determine the curve of the boundary layer more accurately. Other than that, the experiment was fairly successful.

References

[1] Dr. Hamid Rahai, MAE 440 Aerodynamics Laboratory Experiments, California State University Long Beach, Spring 2007

[2] John J. Bertin, Aerodynamics for Engineers, 4th edition, 2002

[3] Schlichting H. 1979. Boundary-layer theory. 7th ed. New York: McGraw-Hill.