Multiple snapshot and multiple frequency compressive matched field processing

Kay L. Gemba, William S. Hodgkiss, and Peter Gerstoft

Marine Physical Laboratory of the
Scripps Institution of Oceanography
University of California at San Diego
gemba@ucsd.edu

Motivation and objectives

Compressive sensing (CS) is useful for resolving coherent multipath in beamforming applications and sparse channel estimation (equalization). CS also might be useful in snapshot deficient scenarios and for non-standard array geometries.

Here, we investigate CS performance in the MFP application and demonstrate:

1. CS is equivalent in tracking performance to the Bartlett processor for a single-source scenario with single and multiple snapshots/frequencies using the row-sparsity constraint.

2. CS behaves similarly to an adaptive processor. The output of CS is compared to the white noise constraint (WNC) processor in a two-source scenario.

- Results are demonstrated with simulated and SwellEx-96 data.

Single snapshot compressive sensing

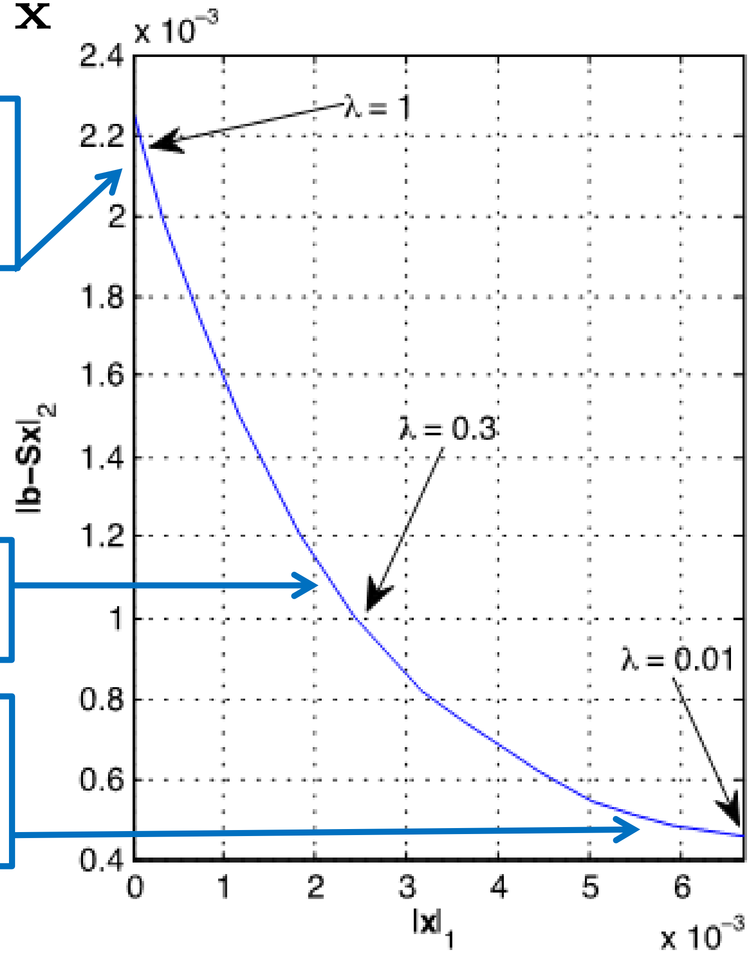
*Convex m*inimization problem using ℓ_1 - norm (basis pursuit)

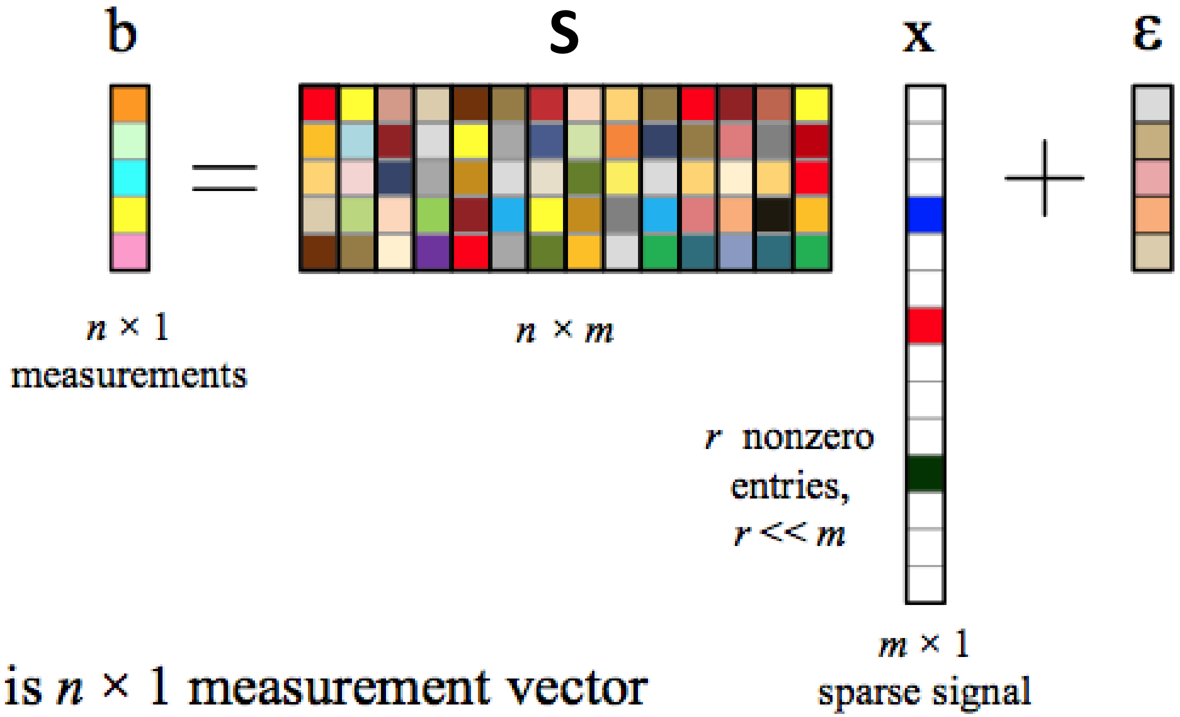
$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} \|\mathbf{b} - \mathbf{S}\mathbf{x}\|_2 + \lambda\|\mathbf{x}\|_1$$

Enforce sparseness by minimizing the second term

λ with minimum overall error

Minimize error term, ignore sparseness



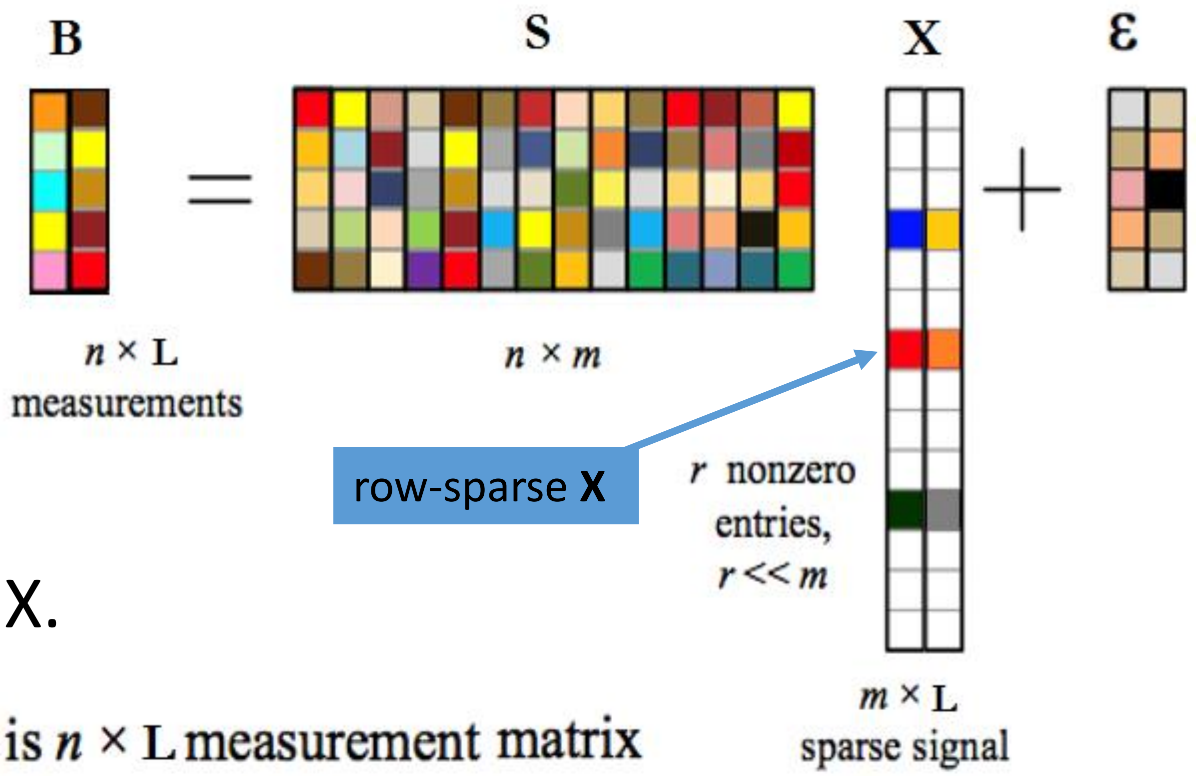


- b is $n \times 1$ measurement vector
- **S** is $n \times m$ measurement/Dictionary matrix, $m >> n$
- x is $m \times 1$ desired vector which is sparse with r nonzero
- ε is the measurement noise

Multiple snapshot compressive sensing

$$\hat{\mathbf{x}} = \underset{\mathbf{X}\in C^{MxL}}{\operatorname{argmin}} \|B - SX\|_F^2 + \lambda\sum_{j=1}^{M}\|X_j\|_2$$

- The row-sparsity constraint is a combination of the L2 and L1 norm.
- The L2 norm operates on the jth row of X.
- Each snapshot solution has its own complex amplitude, unlike conventional MFP (e.g., WNC).



- B is $n \times L$ measurement matrix
- S is $n \times m$ measurement/Dictionary matrix, $m >> n$
- x is $m \times L$ desired matrix which is sparse with r nonzero
- ε is the measurement noise

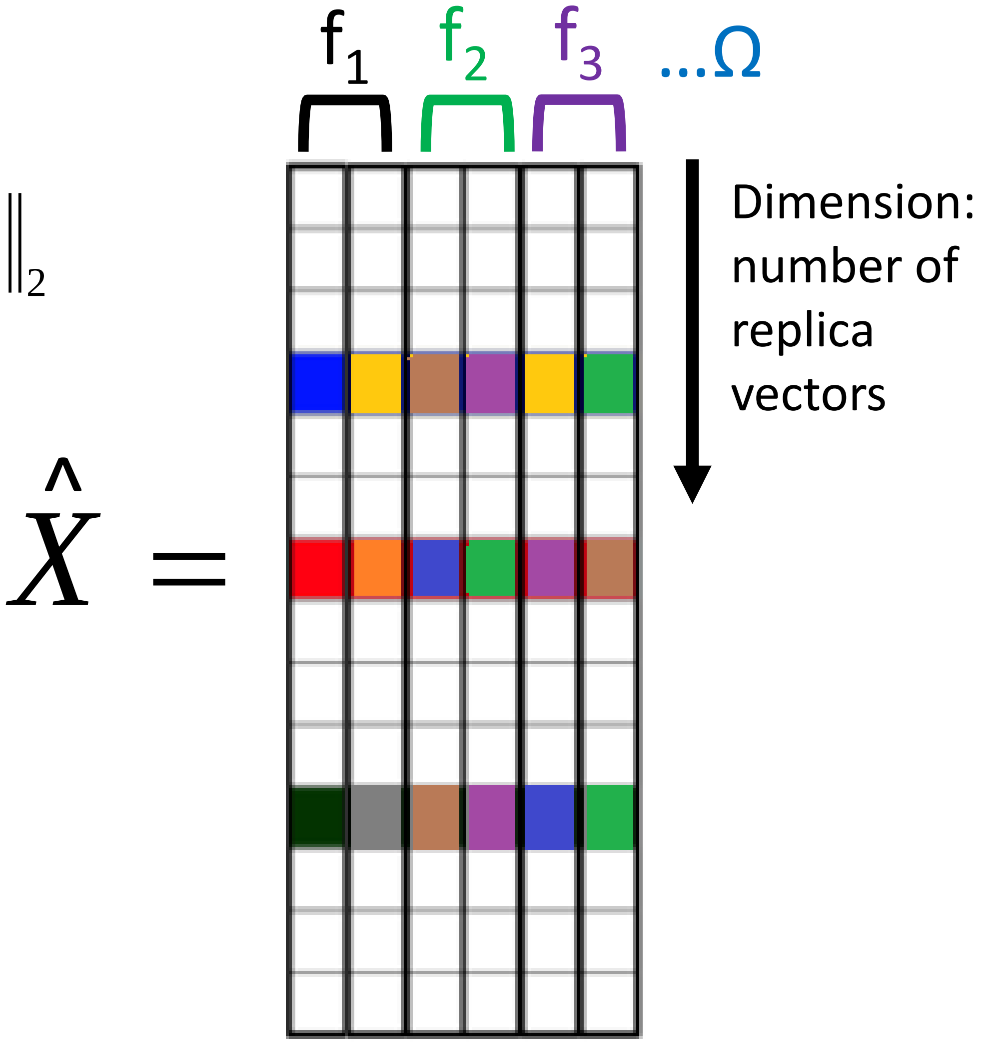
Multiple frequency compressive sensing

Number of frequencies

$$\hat{X} = \underset{\mathbf{X} \in C^{M \times (L\Omega)}}{\arg\min} \sum_{i=1}^{\Omega} \left\| B(f_i) - S(f_i) X(f_i) \right\|_F^2 + \lambda \sum_{j=1}^{M} \left\| X_j \right\|_2$$

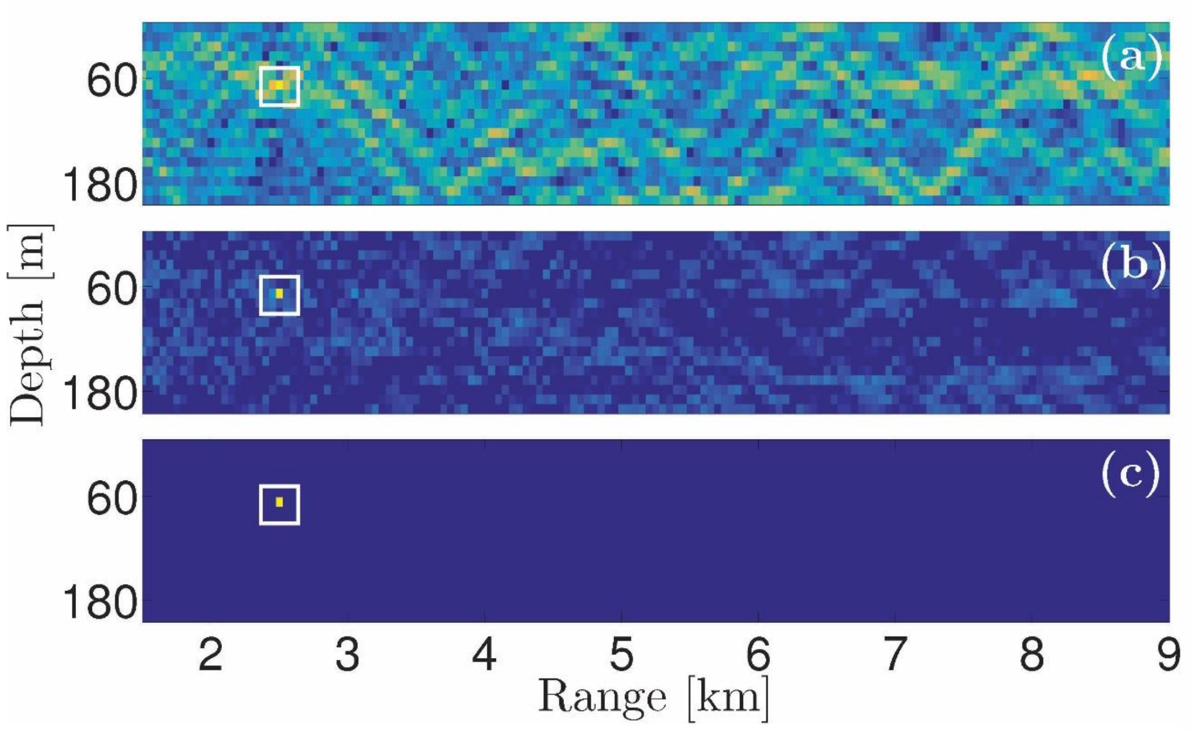
$$X = [X(f_1) X(f_2), \ldots, X(f_\Omega)]$$

- Multi-frequency incoherent cost function subject to the row-sparsity constraint.
- The row-sparsity constraint enforces the same sparsity for all frequency snapshots in X.

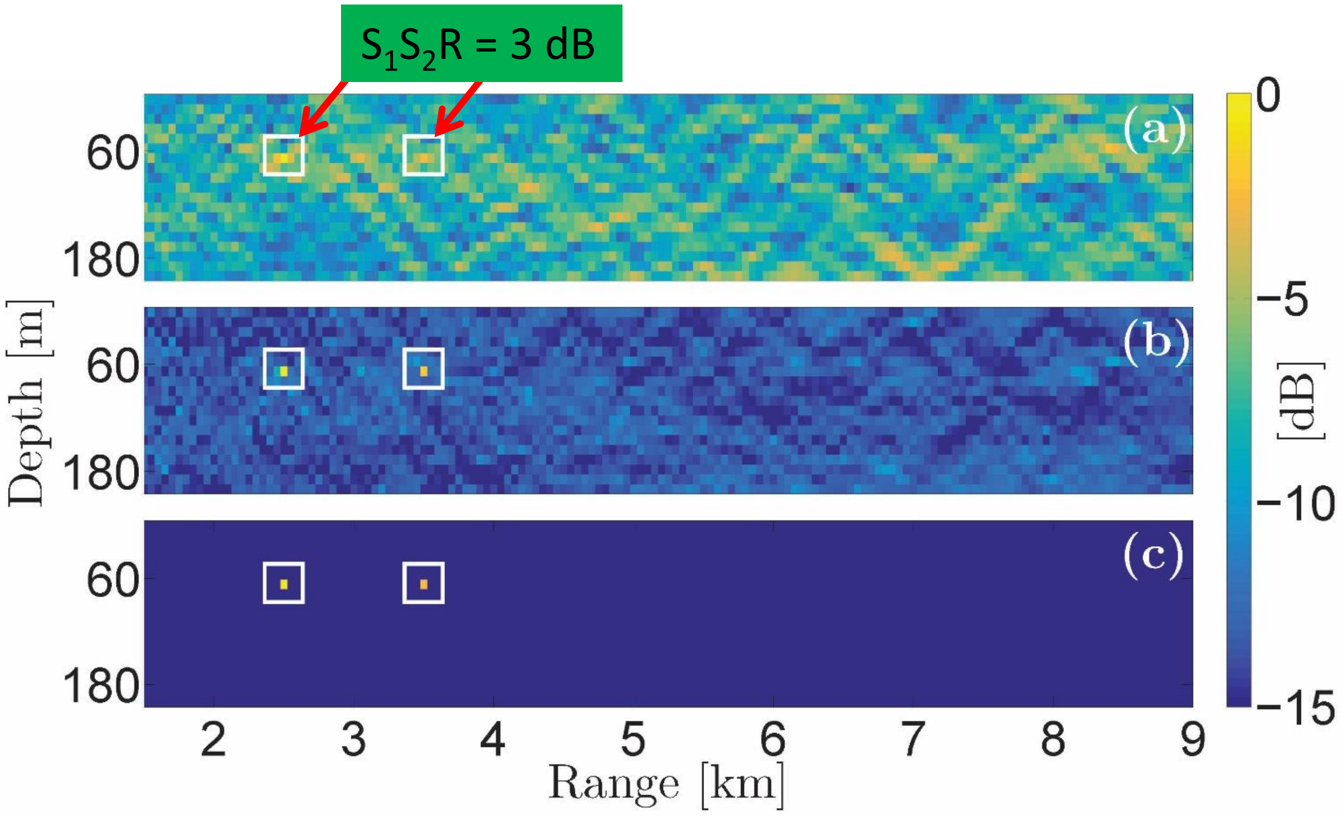


Example: 3 frequencies, 2 snapshots each, 3 row-sparse solutions

SNR Localization Curves – Simulation Intro



Single source localization simulation at SNR 0 dB and 166 Hz: (a) Bartlett, (b) WNC -6 dB, and (c) CS. True source locations are marked by white squares and all processors localize the single source.

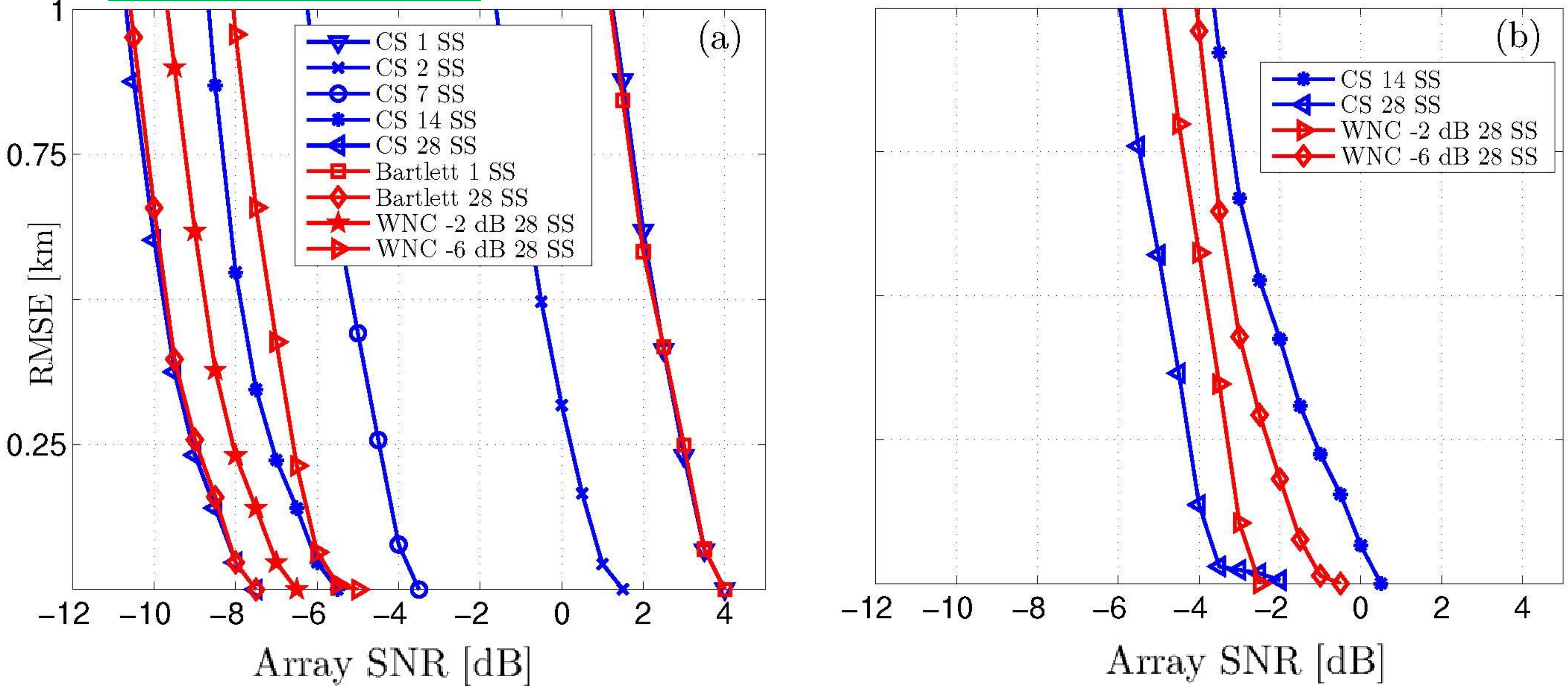


Incoherent 2 source localization simulation at SNR 0 dB and 166 Hz: (a) Bartlett, (b) WNC -6 dB, and (c) CS. Bartlett has several competing sidelobes at higher levels than Source 2. WNC and CS localize both.

Single source

Single Frequency

Two sources

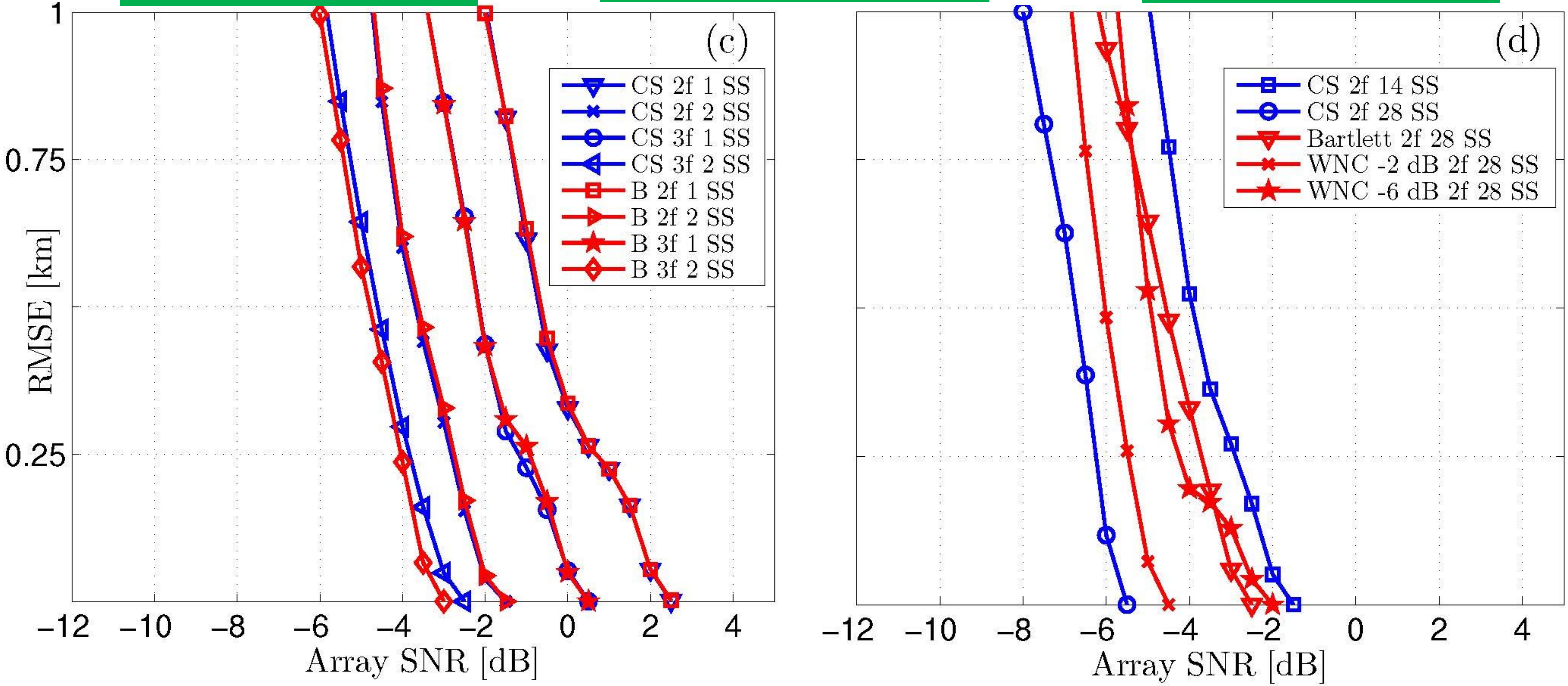


Root mean square error (RMSE) localization performance for single and multiple snapshots (SS).
Single frequency 166 Hz panels show (a) Source 1 only and (b) Sources 1 and 2.
In Panel (b), SNR and RMSE correspond to Source 2.

Single source

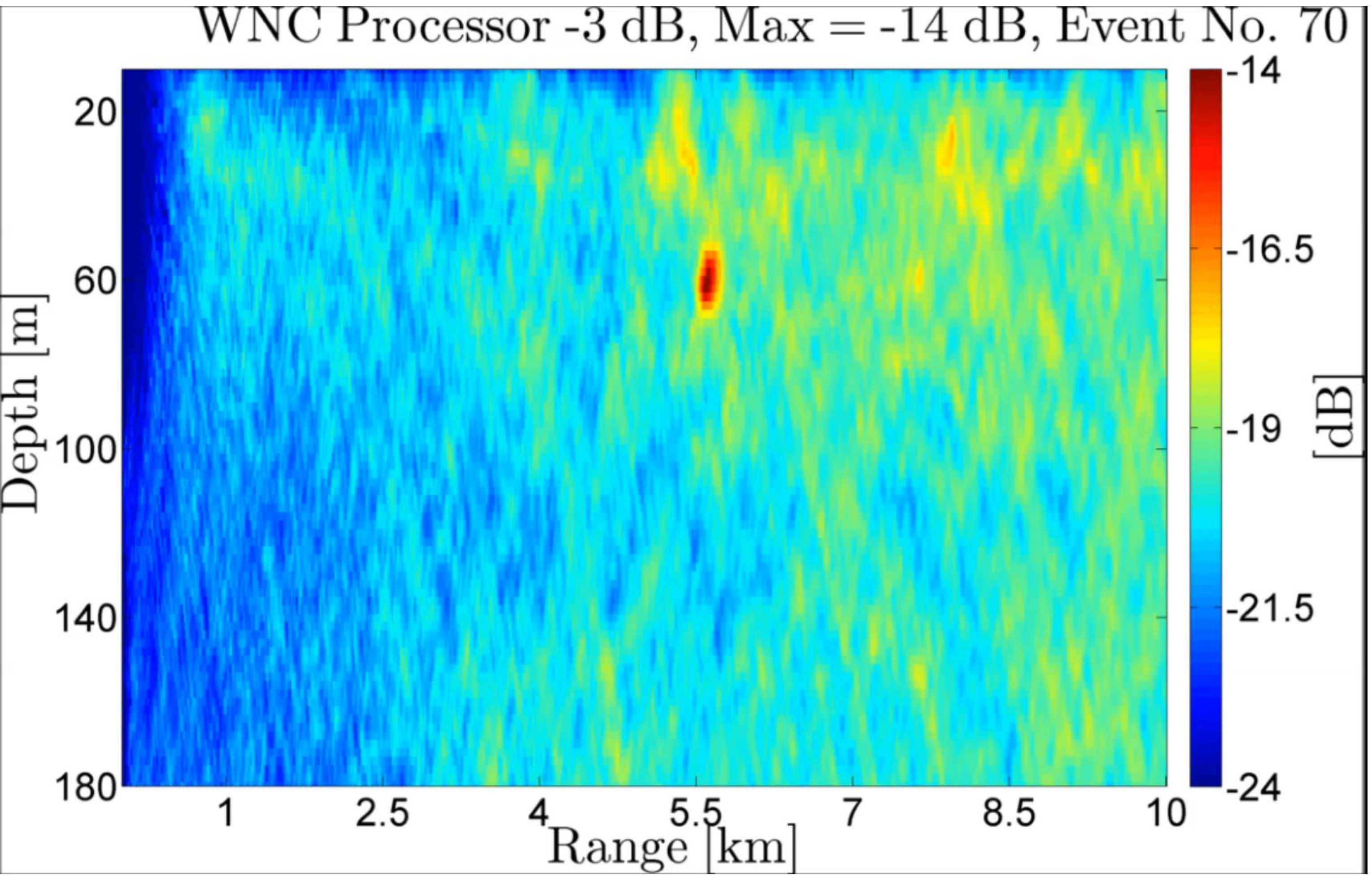
Multi-Frequency

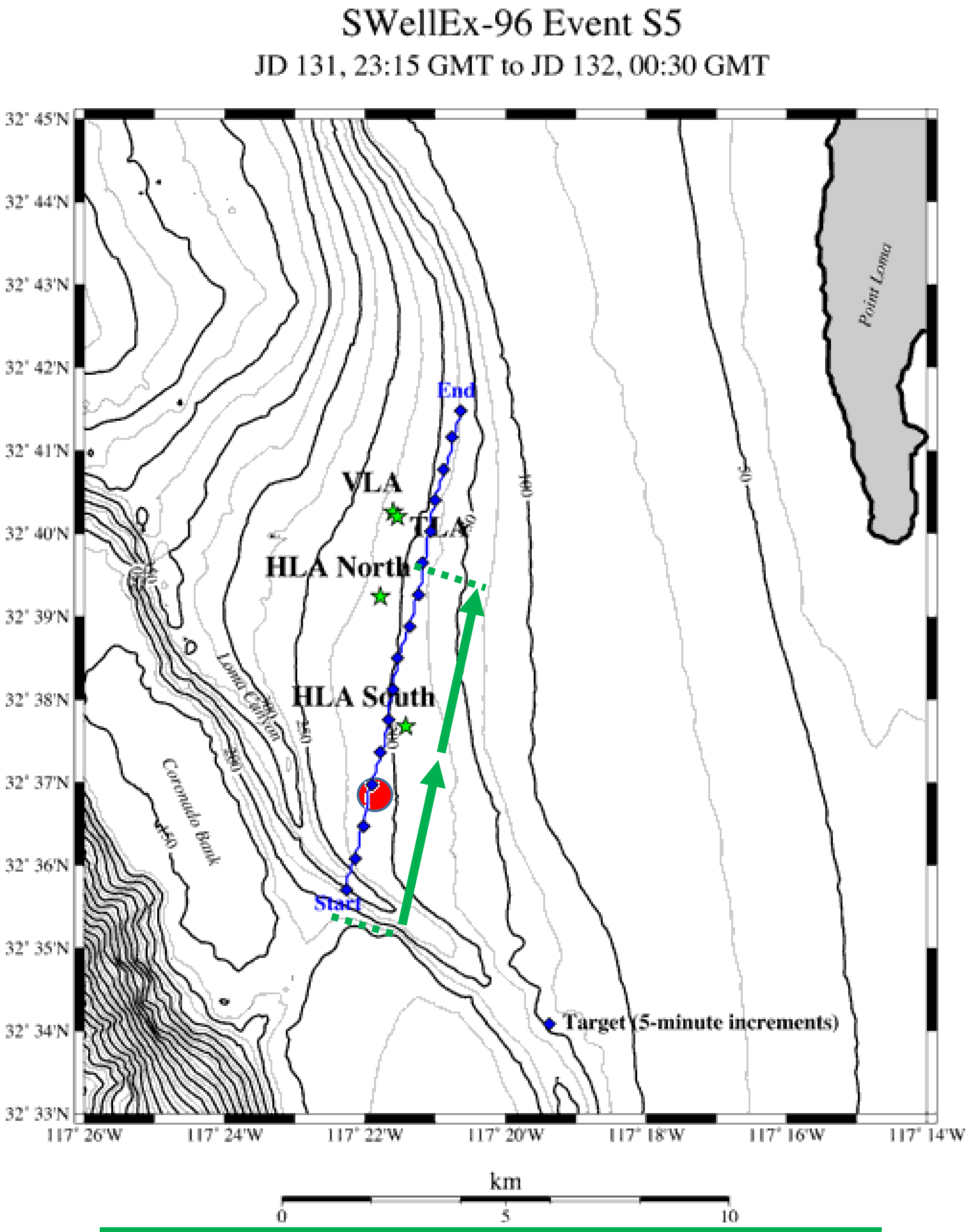
Two sources



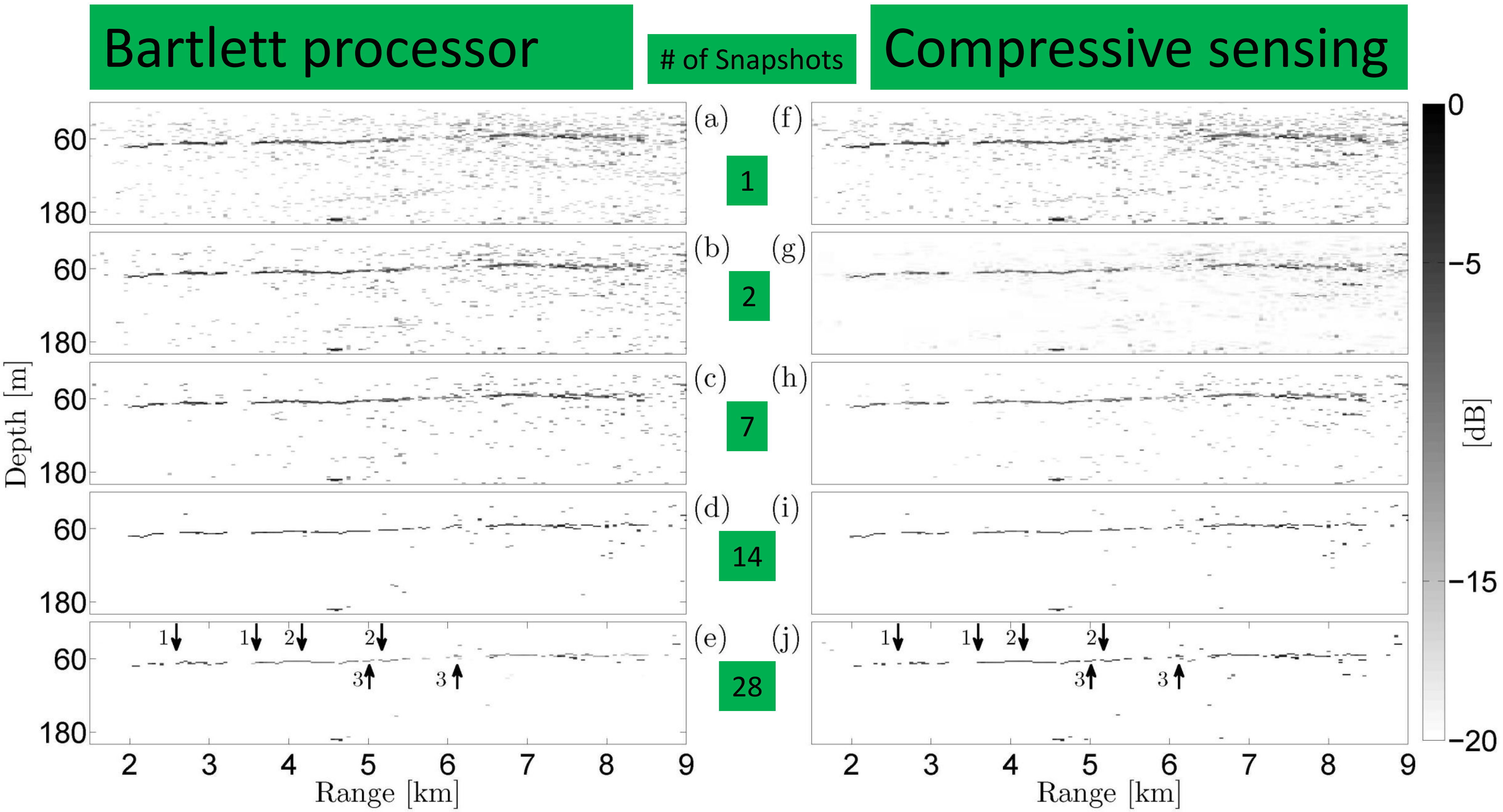
Multi-frequency panels show (c) Source 1 only and (d) Sources 1 and 2. The two sets of frequencies are 2f = 148, 166 Hz and 3f = 148, 166, 235 Hz. In Panel (d), SNR and RMSE correspond to Source 2.

SWellEx-96 – Event S5 – Deep Source

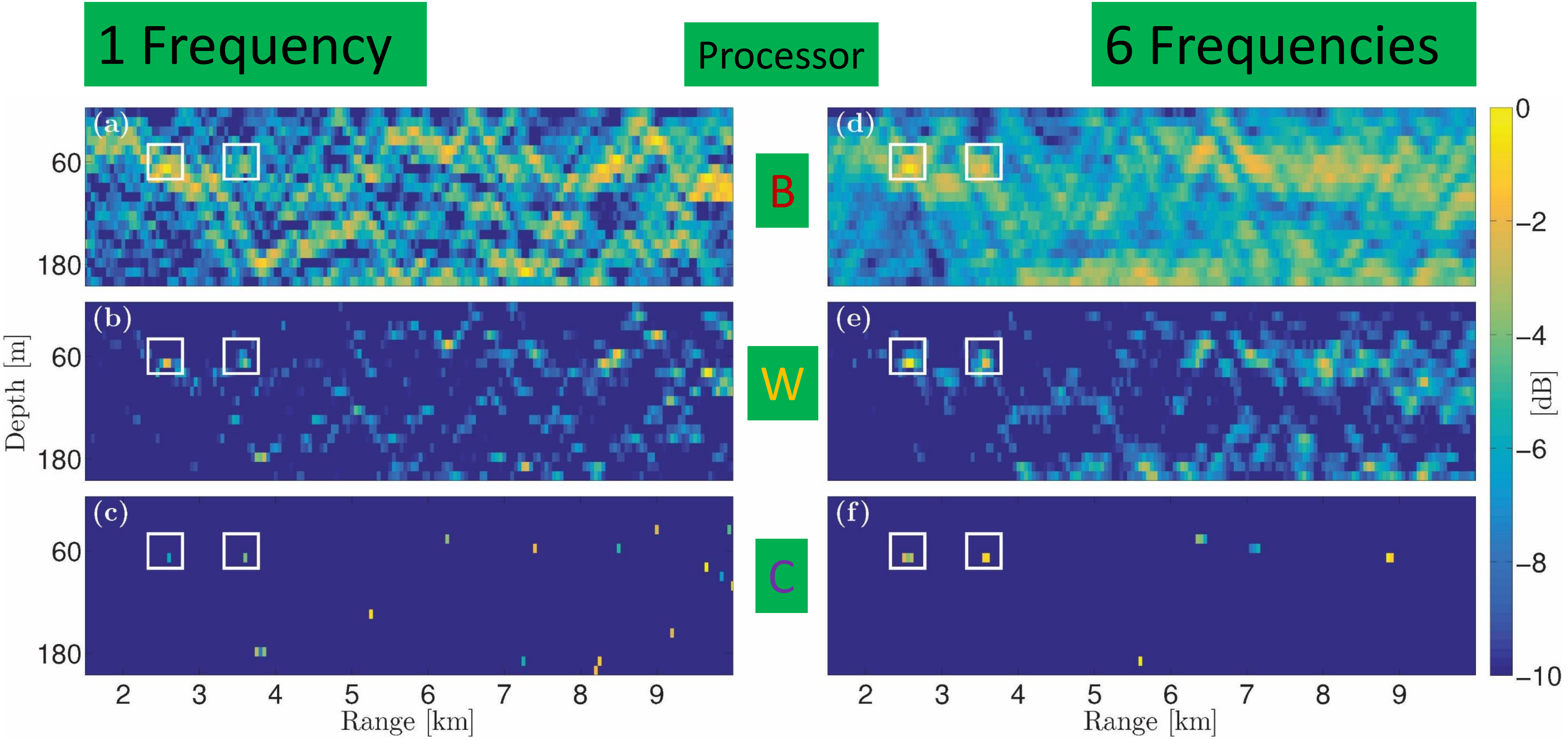




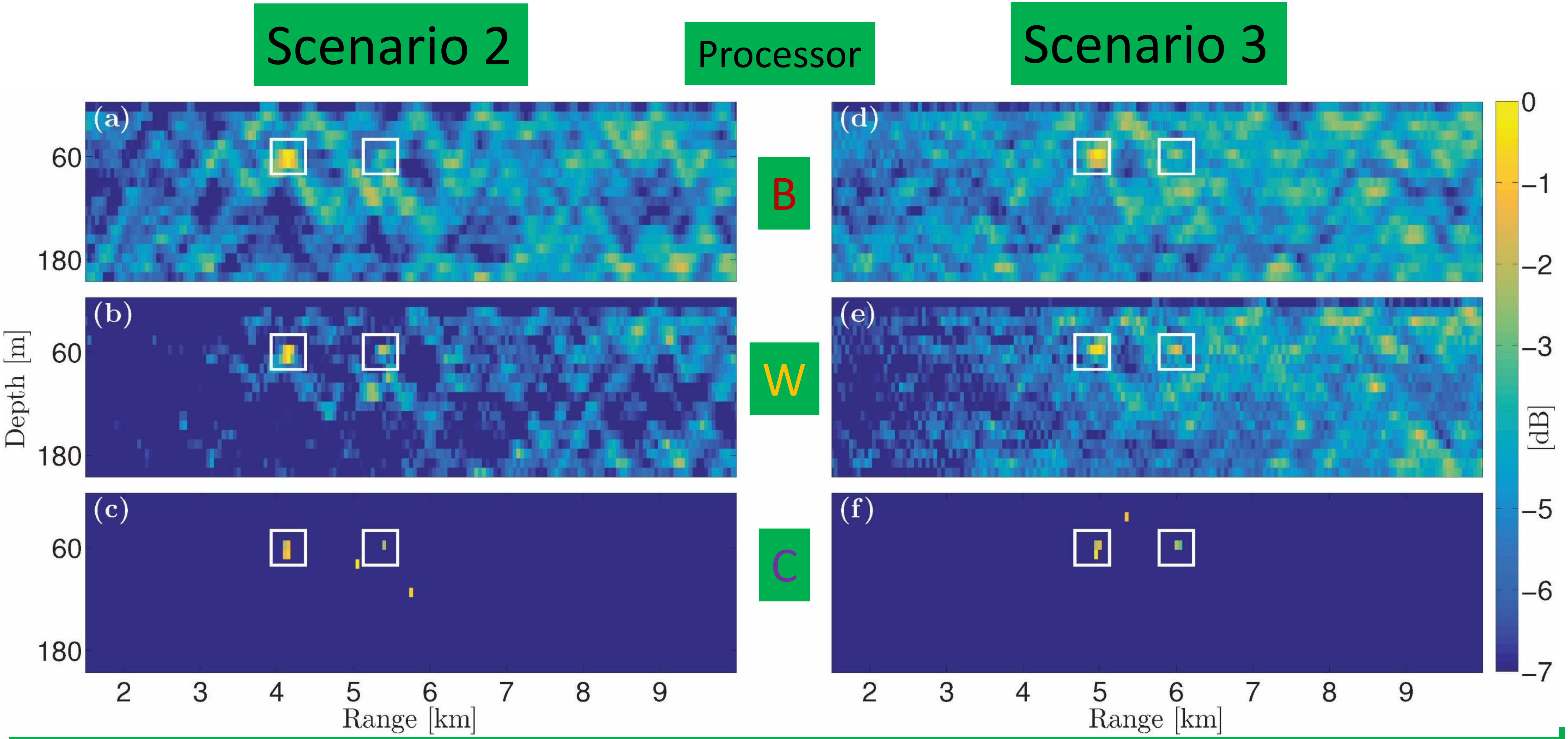
The green track indicates the selected part of the data for analysis



Bartlett processor
# of Snapshots
Compressive sensing
(a)
(f)
1
(b)
(g)
2
(c)
(h)
7
(d)
(i)
14
(e)
(j)
28
Depth [m]
60
180
Range [km]
2 3 4 5 6 7 8 9
[dB]
0
−5
−15
−20



Scenario 1. True positions are indicated by white squares. Panels on the left use a single frequency (166 Hz), panels on the right use 6 frequencies (94; 112; 130; 148; 166; 235 Hz): (a,d) Bartlett, (b,e) WNC -2 dB, (c,f) CS. Incorporating multiple frequencies reduces ambiguity and helps localize Source 2.



Localization results using 6 frequencies (94; 112; 130; 148; 166; 235 Hz). True positions are indicated by white squares. Left panels show Scenario 2, right panels show Scenario 3 for: (a,d) Bartlett, (b,e) WNC -2 dB, (c,f) CS. Bartlett displays the most ambiguity while WNC and CS exhibit good performance with few false localizations.

Conclusions

- CS behaves similarly to an adaptive processor and can discriminate against sidelobes. For the matched field processing application, CS is comparable to the performance of the WNC processor.
- CS and Bartlett tracking yield identical localization results for a single source using multiple snapshots and multiple frequencies.
- CS (using the row-sparsity constraint) appears robust to modest data-replica mismatch and situations when multiple snapshots correspond to adjacent range-depth cells at the expense of possible additional solutions.

Gemba K L, Hodgkiss W S and Gerstoft P. Adaptive and compressive matched field processing, J Acoust. Soc. Am., submitted April 2016.

- End of presentation -